



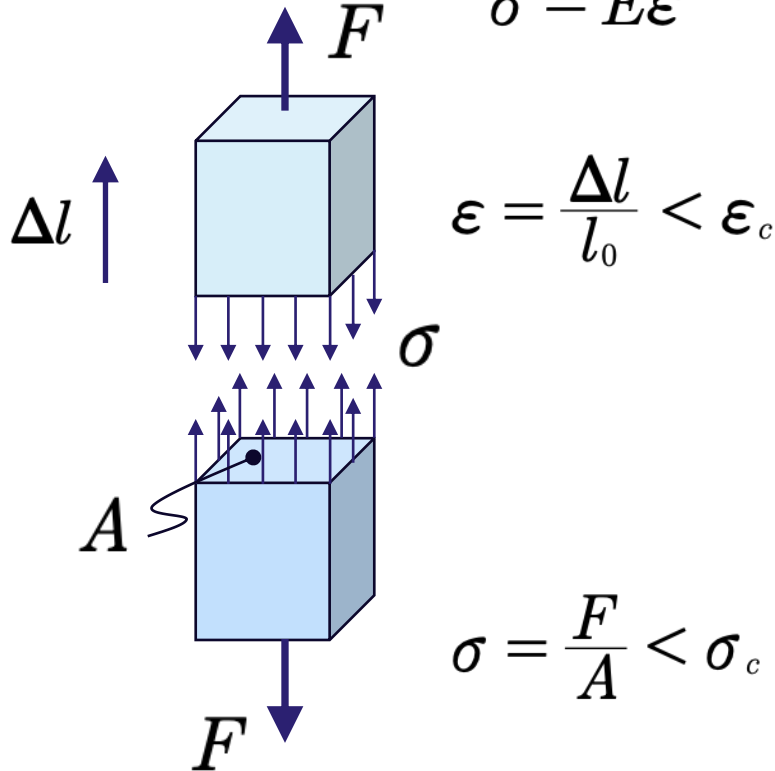
Introduction to fracture mechanics

Gergely Molnár



Theoretical basics

Mechanical design



1. Idealize the structure
2. Define loading & boundary conditions
3. Define material behavior
4. Calculate the mechanical response
5. Identify critical locations & failure modes
6. Apply failure/design criteria
7. Size the component
8. Verify & iterate

Theoretical basics

Example

Paper

- Geometry

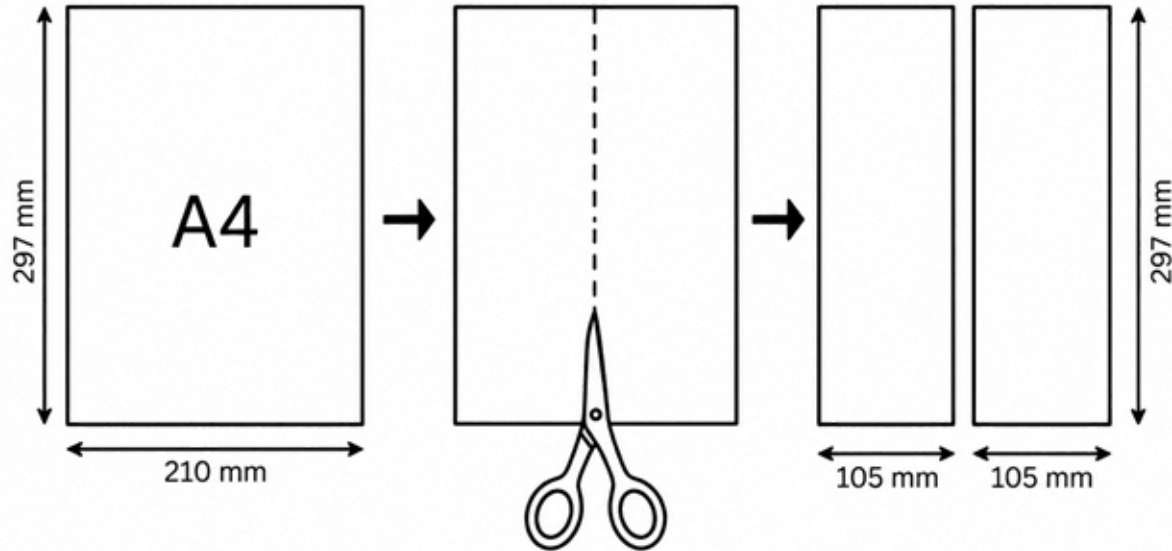
$$L = 105 \text{ mm}$$

$$t = 0.1 \text{ mm (75 g/m}^2\text{)}$$

$$A = L \cdot t = 10.5 \text{ mm}^2$$

- Material property

$$\sigma_c = 30 \text{ MPa}$$

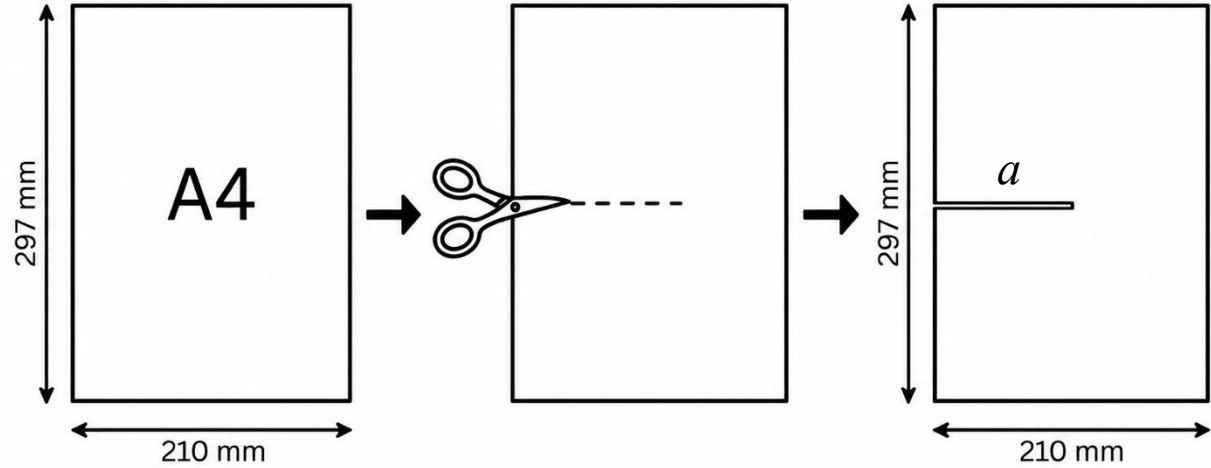


$$F_c = A \cdot \sigma_c = 315 \text{ N} \approx 32 \text{ kg}$$



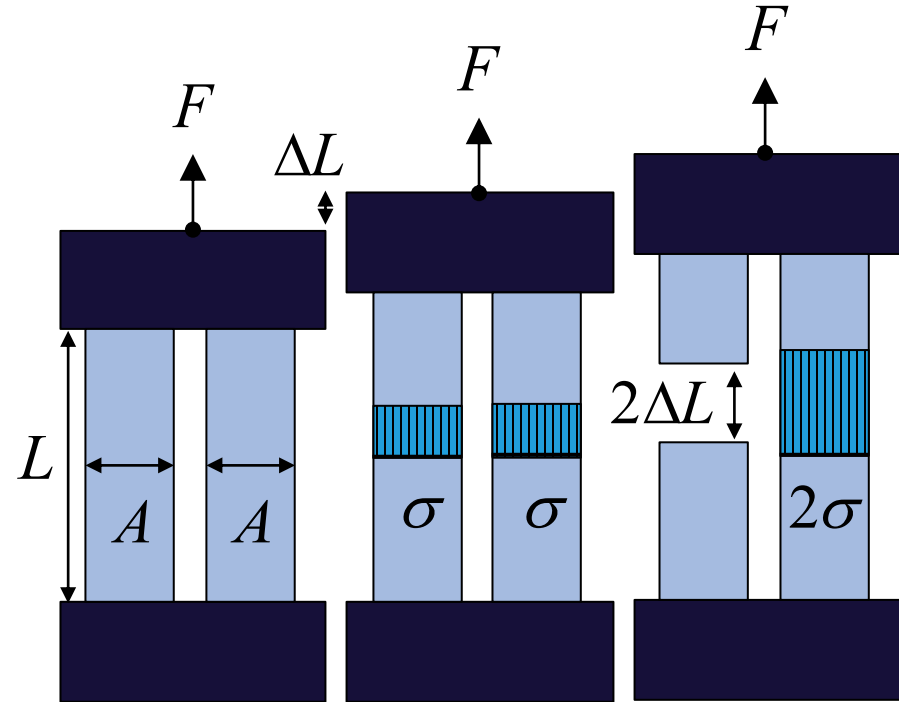
Theoretical basics

Example Paper



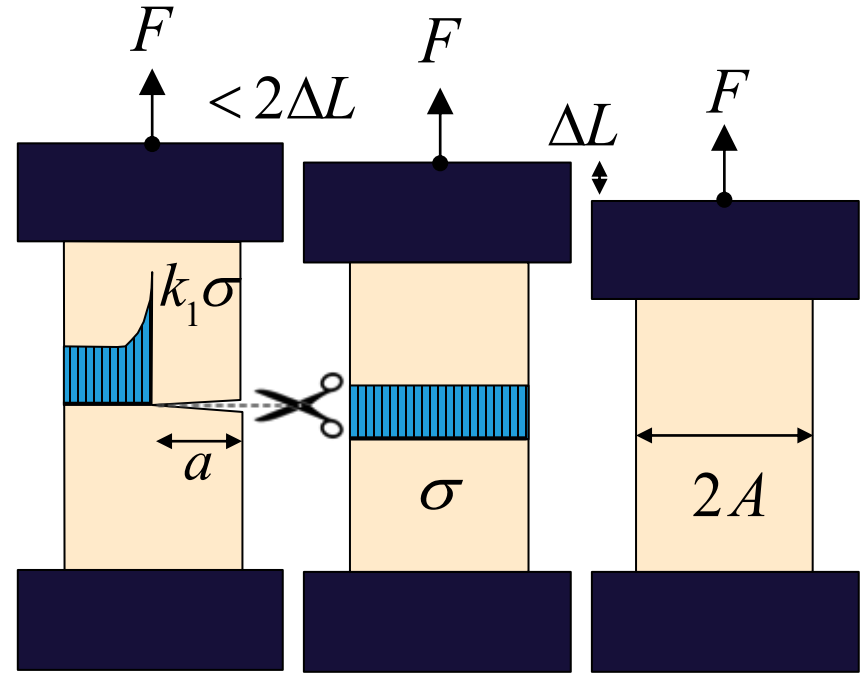
Theoretical basics

Classical mechanics



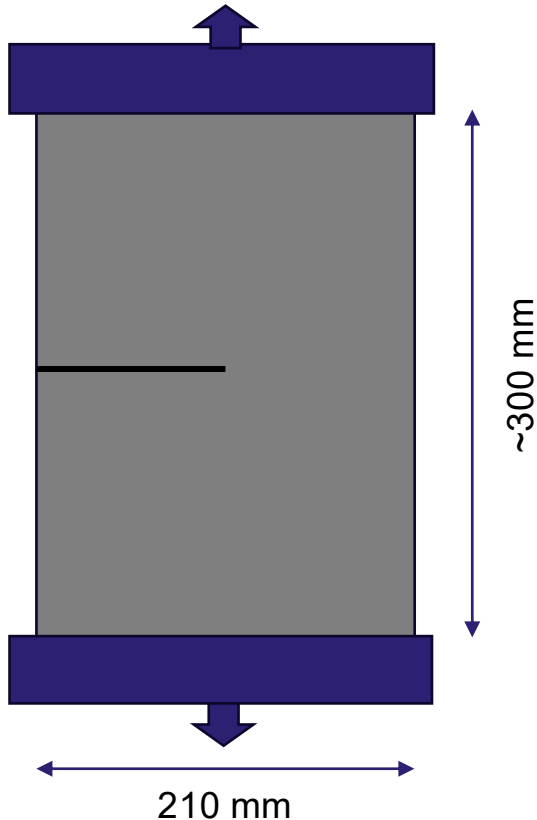
$$\sigma = \frac{F}{2A}$$

Fracture mechanics



Theoretical basics

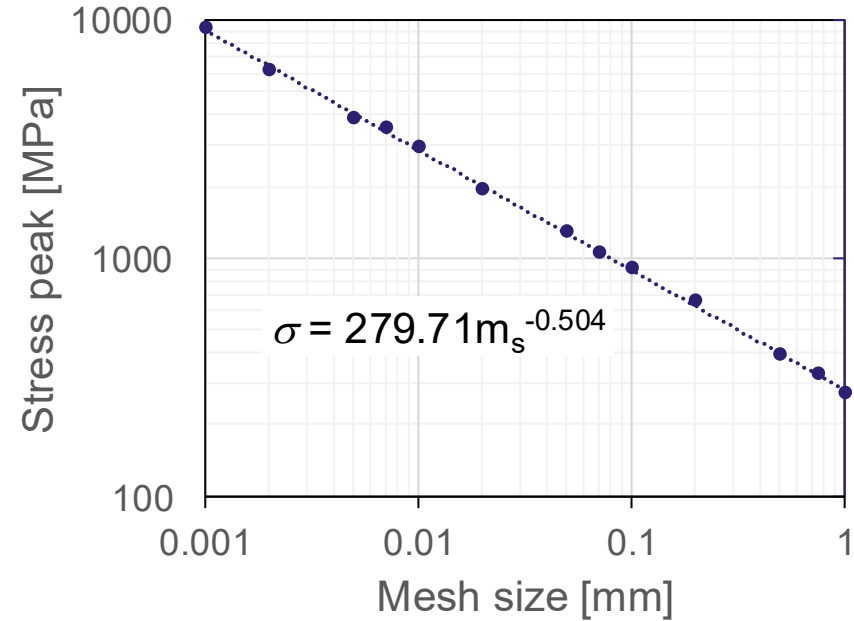
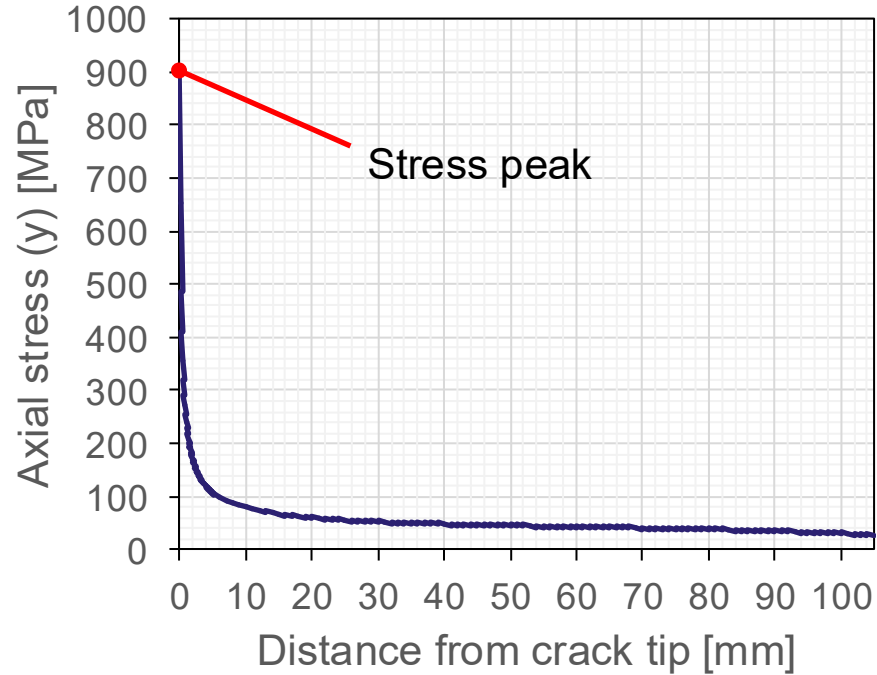
Finite element



1. Idealize the structure
2. Define loading & boundary conditions
3. Define material behavior
4. Calculate the mechanical response
 - a) Convergence analysis
5. Identify critical locations & failure modes
6. Apply failure/design criteria
7. Size the component
8. Verify & iterate

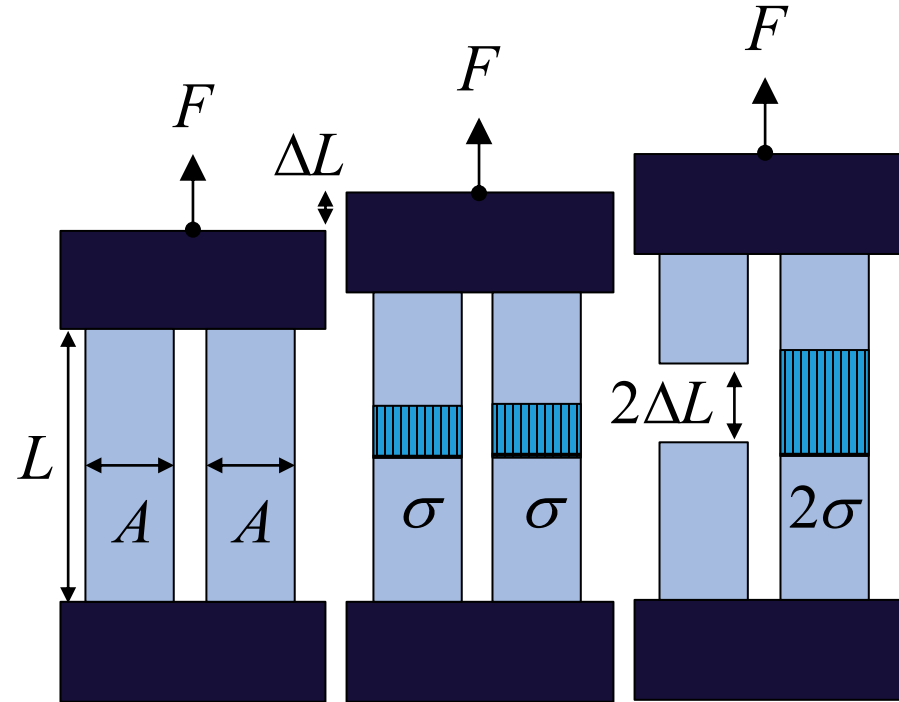
Theoretical basics

Stress singularity



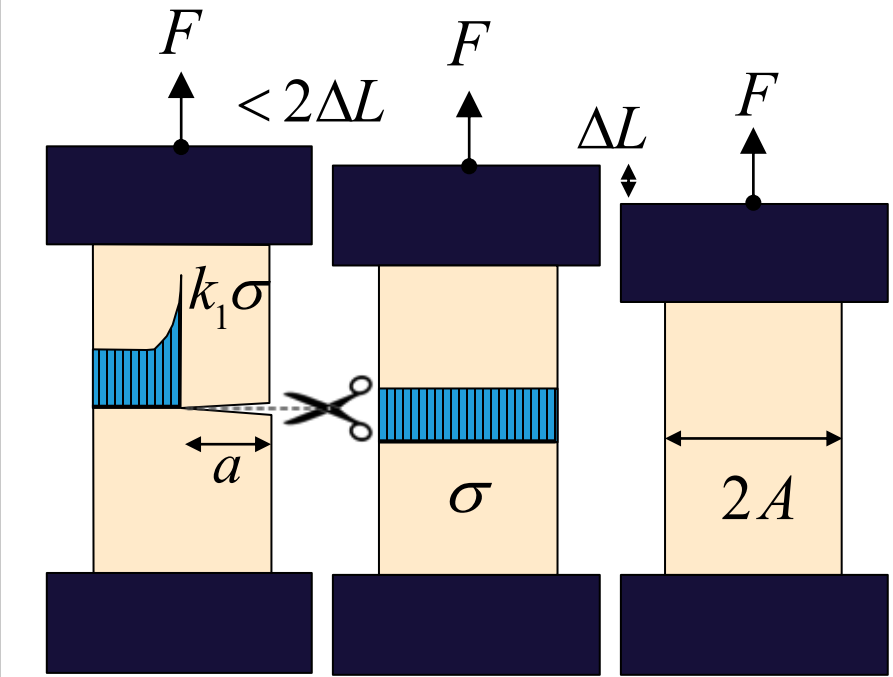
Theoretical basics

Classical mechanics



$$\sigma = \frac{F}{2A}$$

Fracture mechanics



$$\sigma = \frac{F\sqrt{\pi a}}{2A} \frac{1}{\sqrt{2\pi r}} g(\theta)$$

Theoretical basics

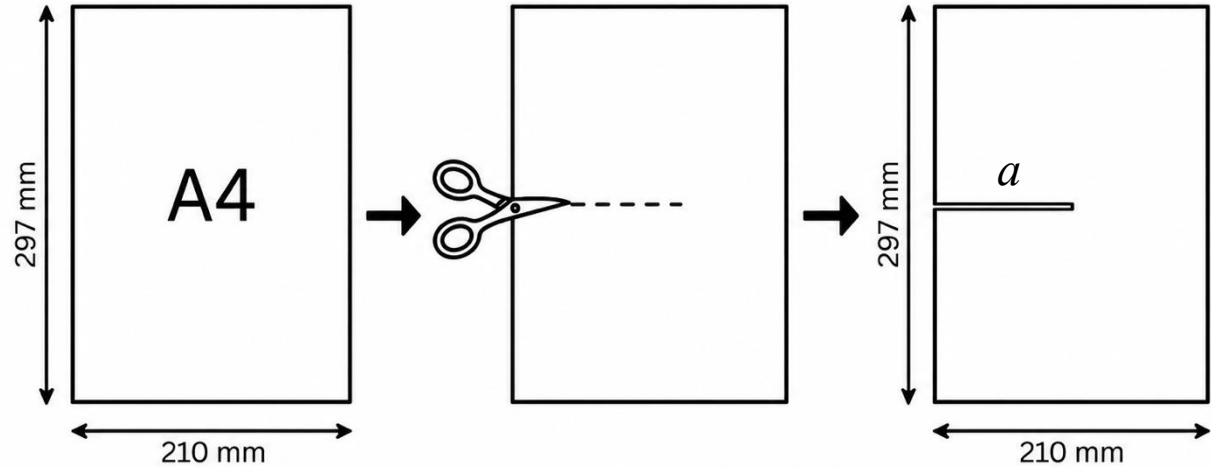
Example

Paper

$$L = 210 \text{ mm}$$

$$t = 0.1 \text{ mm (75 g/m}^2\text{)}$$

$$a = 105 \text{ mm}$$



$$K_I = \frac{F}{L \cdot t} \sqrt{\pi a} \left[1.122 - 0.231 \left(\frac{a}{L} \right) + 10.55 \left(\frac{a}{L} \right)^2 - 21.71 \left(\frac{a}{L} \right)^3 + 30.382 \left(\frac{a}{L} \right)^4 \right]$$

$$K_{IC} = 3.2 \text{ MPa} \sqrt{\text{mm}}$$



$$F_c = 1.3 \text{ N} \approx 130 \text{ g}$$

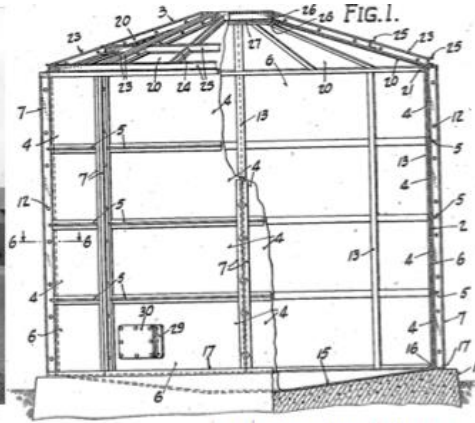
History

Why Failure?

Boston Molasses Disaster, 1919



Matthew M. Yospin © 2018
@MatthewYospin yospin/home



Photograph is in the public domain,
patent illustration is from US 2395685

- ❑ 9 million liter of molasses
- ❑ 15 m high and about 27 m in diameter
- ❑ was leaking molasses

3 years



- ❑ 21 people died
- ❑ 150 injured
- ❑ buildings, railway infrastructure damaged and destroyed



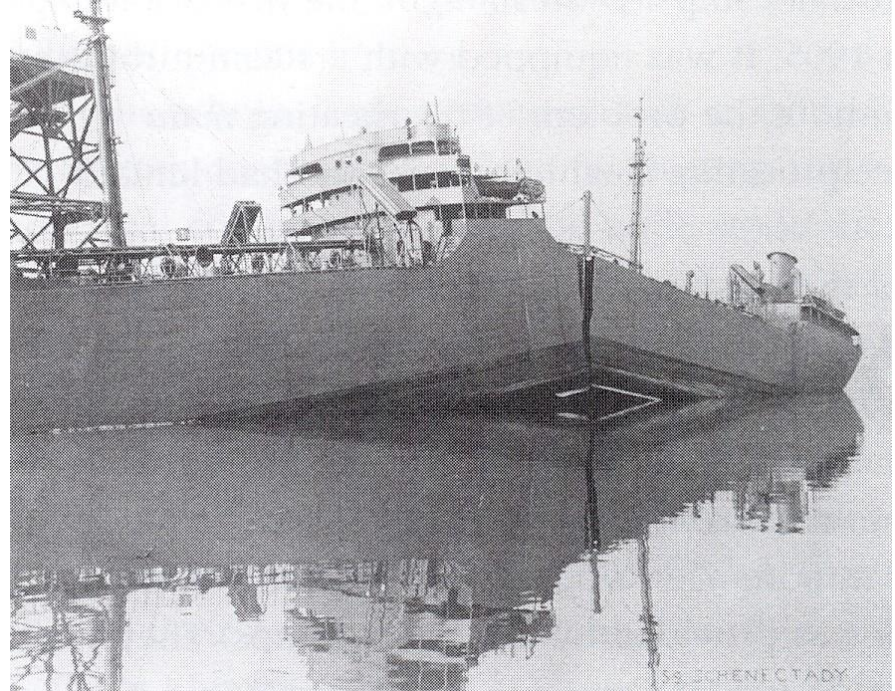
History

Why Fracture?

- ❑ WWII Tanker
- ❑ Hull cracked almost in half
- ❑ Poor quality welds
- ❑ Low-grade steel components
- ❑ Cold temperatures of war-time operations



SS Schenectady, 1943



History

Why Fracture?

- ❑ Catastrophic, unforeseen, devastating failure
- ❑ Overengineering
- ❑ Cost of failure **\$119B/yr** in 1982 ($\approx 4\%$ of GDP of USA)

de Havilland Comet, 1958



Aloha Airlines Flight 243, 1988

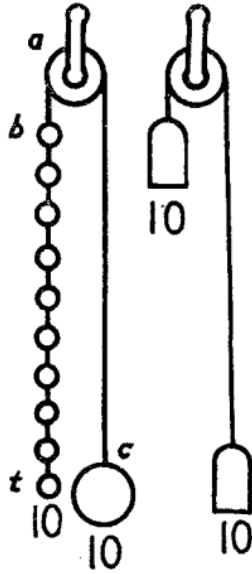
History

History

1500's



(Codex Atlanticus, Leonardo da Vinci)

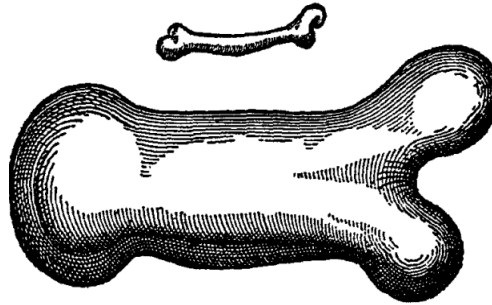


“among cords of equal thickness the **longest** is the **least strong**...”

“a cord is so much **stronger**...as it is **shorter**”

1638

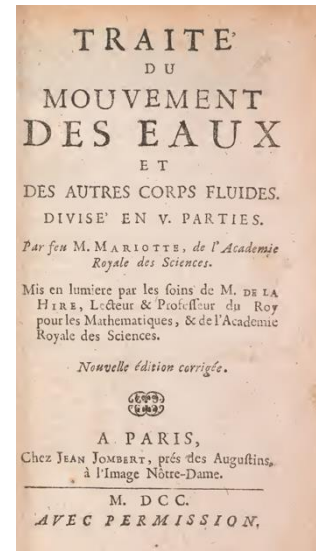
(Two New Sciences, Galileo Galilei)



“...the **smaller** the body the **greater** its relative **strength**”

(Traité du mouvement des eaux, Edmé Mariotte)

1686



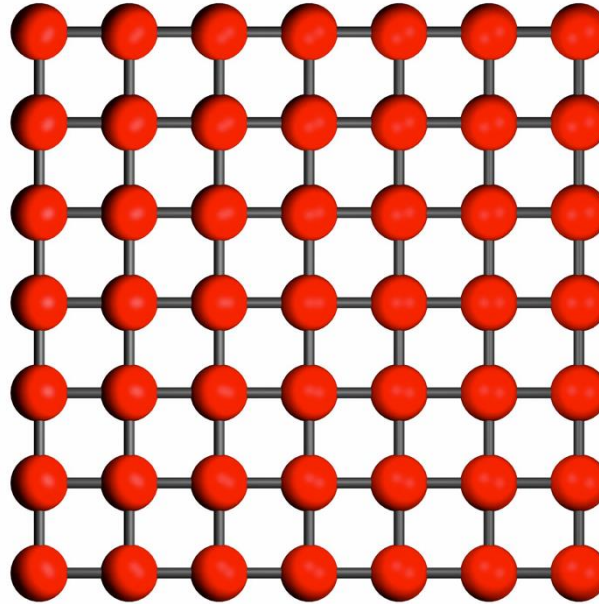
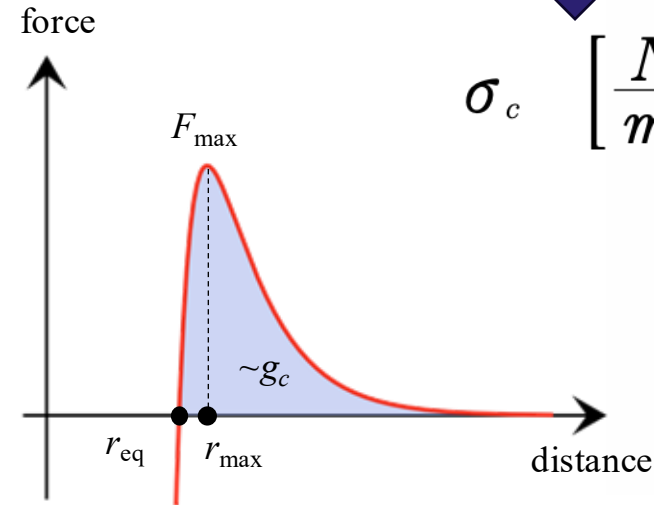
“A long rope and a short one always support the same weight, unless that in a long rope there may happen to be some **faulty place**, in which it will break sooner than in a shorter.”

History

Energy release rate / Toughness

$$g_c \left[\frac{J}{m^2} \right] = \left[\frac{Nm}{m^2} \right] = \left[\frac{N}{m} \right]$$

$$\sigma_c \left[\frac{N}{m^2} \right]$$



Griffith (1921, 1924)



History

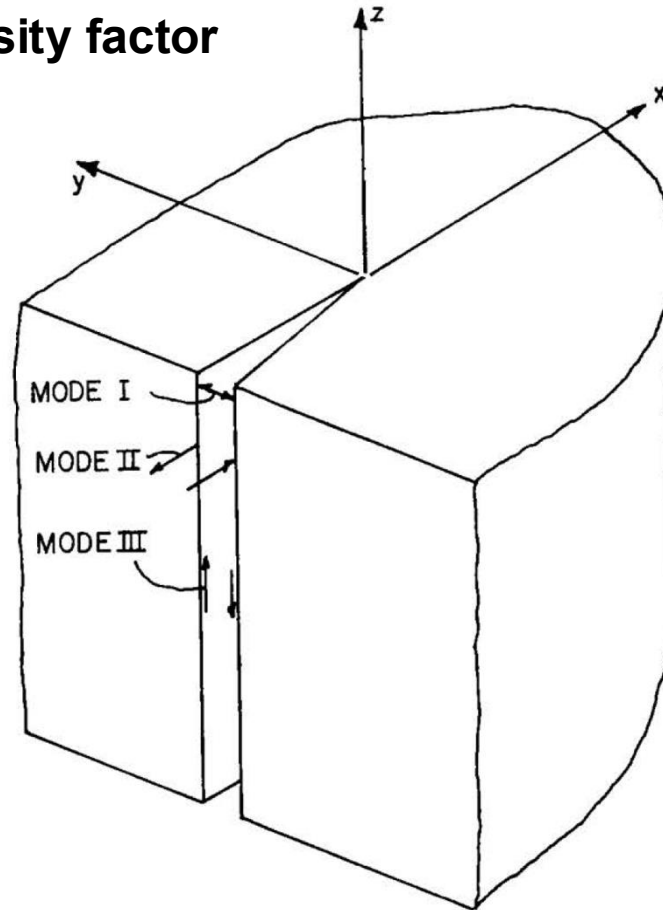
Modes of fracture / Stress intensity factor

$$\sigma_{ij}(r, \theta) = \sum_{k=I,II,III} \frac{K_k}{\sqrt{2\pi r}} f_{k,ij}(\theta)$$

$$K = \lim_{r \rightarrow 0} \sqrt{2\pi r} \sigma_{y,xy,yz}(r, \theta = 0)$$

$$K \leq K_c \quad \text{Pa}\sqrt{\text{m}}$$

$$G = \sum_{k=I,II,III} K_k^2 g(E, \nu)$$

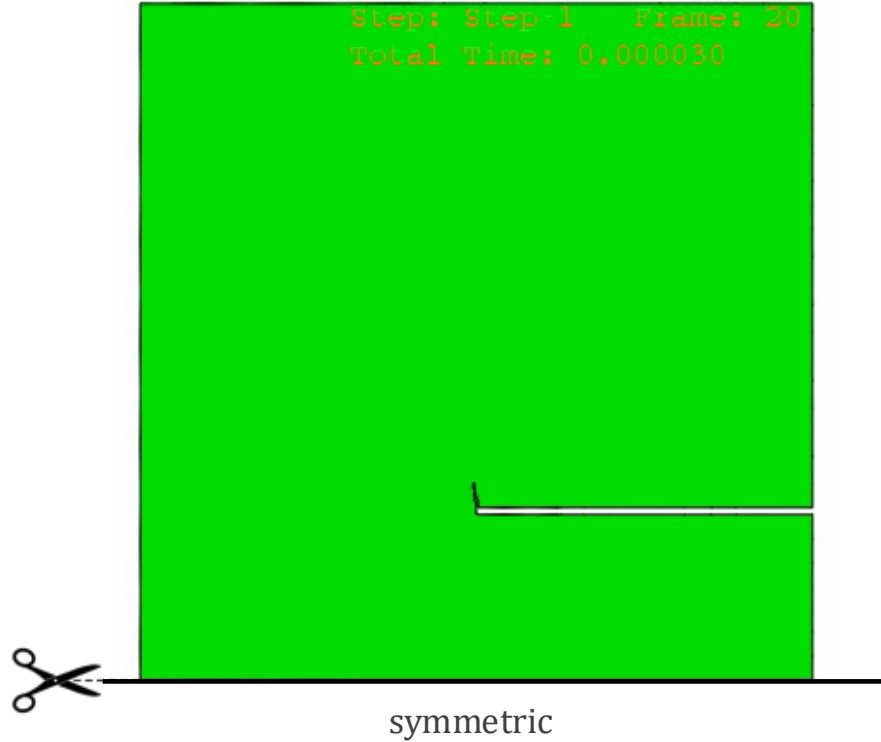


Irwin (1967, 1958)
McClintock & Irwin (1968)

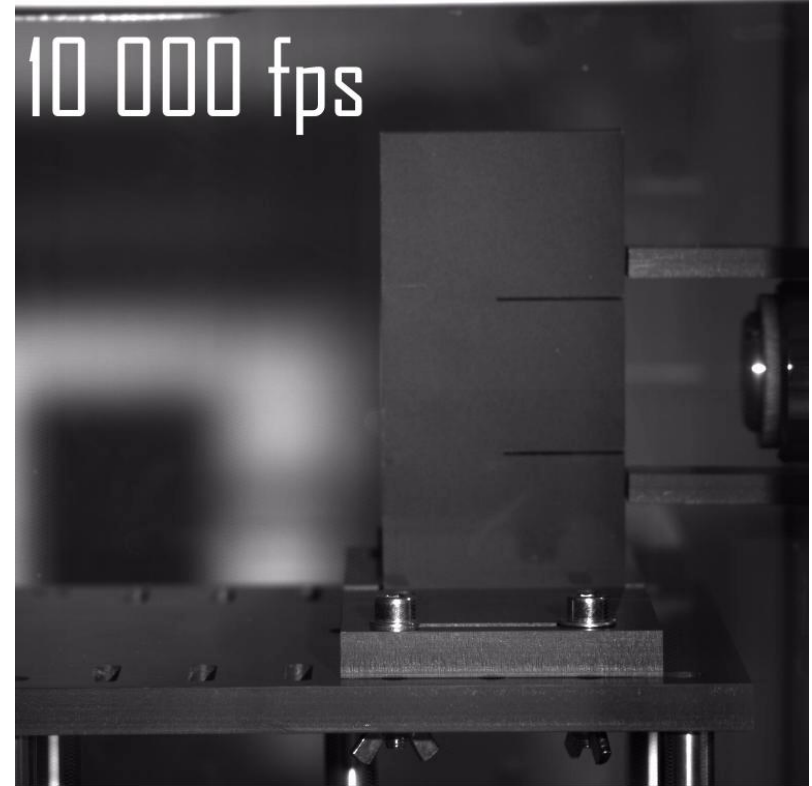


History

Simulation



Experiments

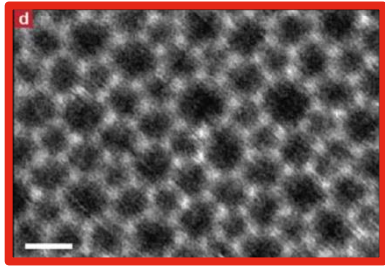
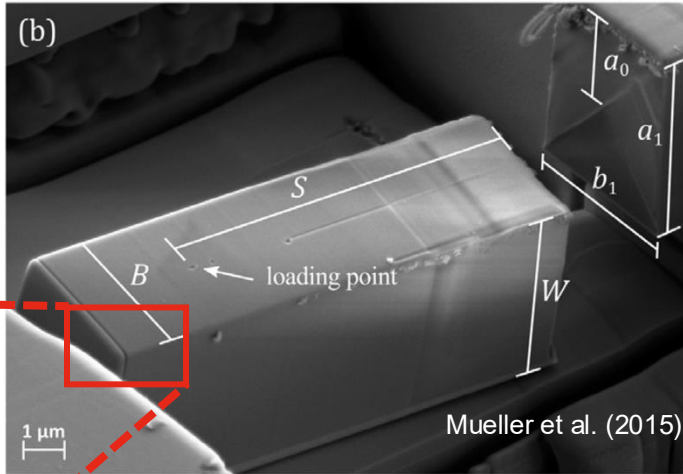
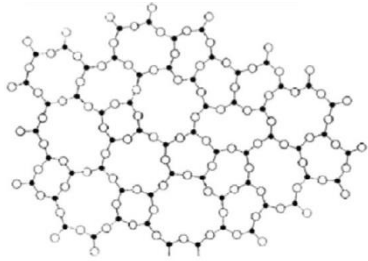


History

Exploring the microscopic scale

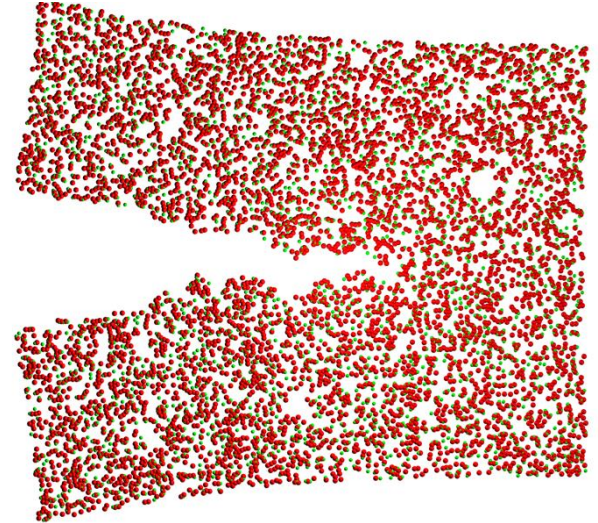
Micron scale experiments

Zachariasen (1932)



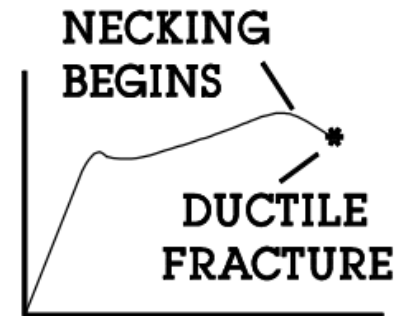
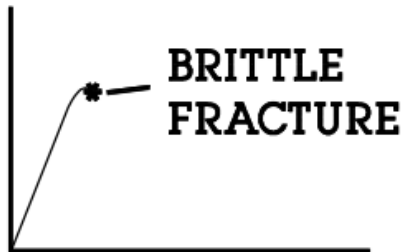
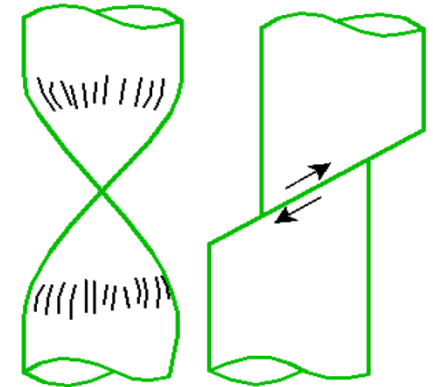
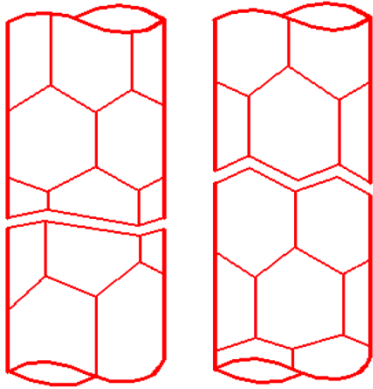
Huang et al. (2012)

Atomic scale simulations



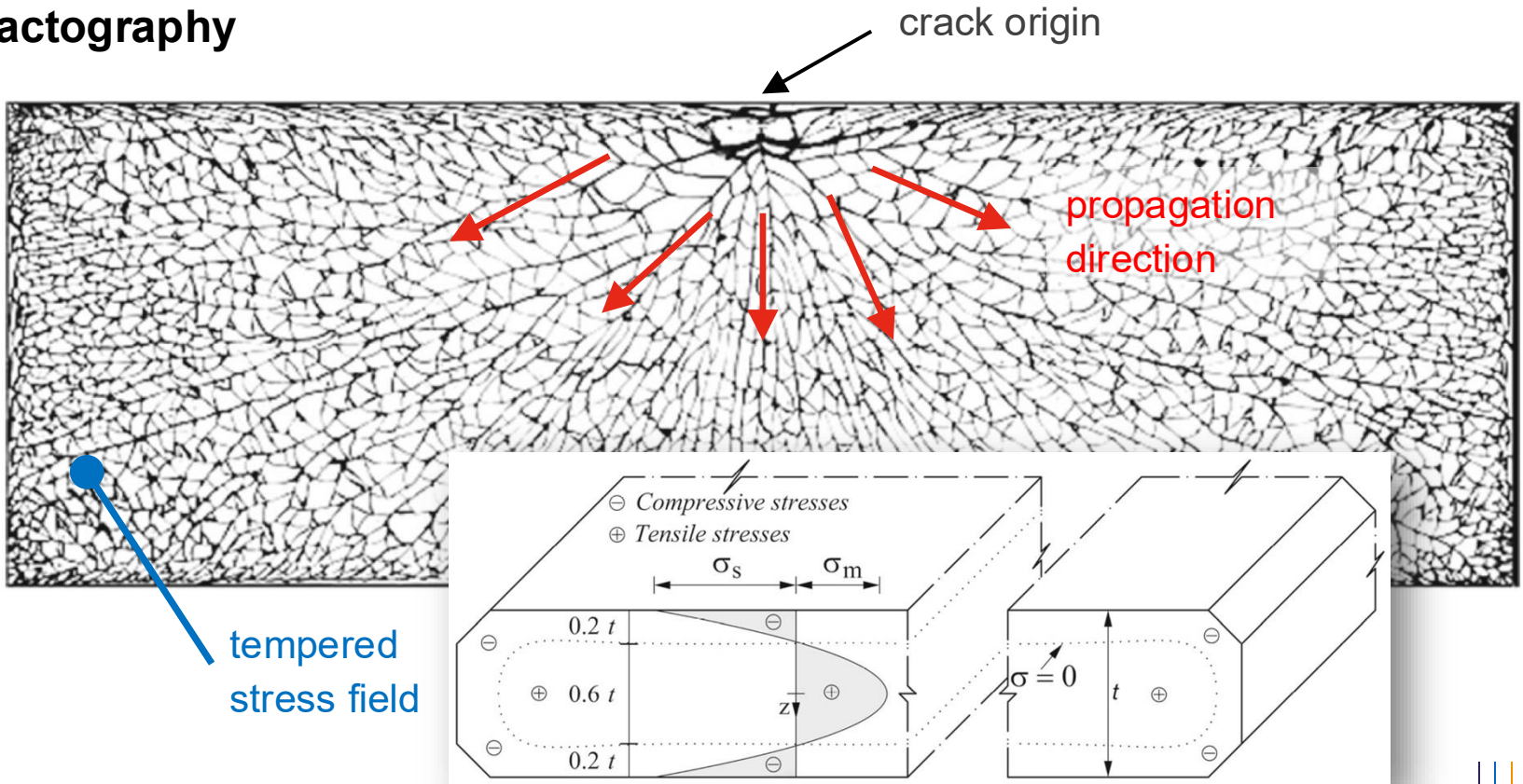
Types of fracture

Brittle / ductile



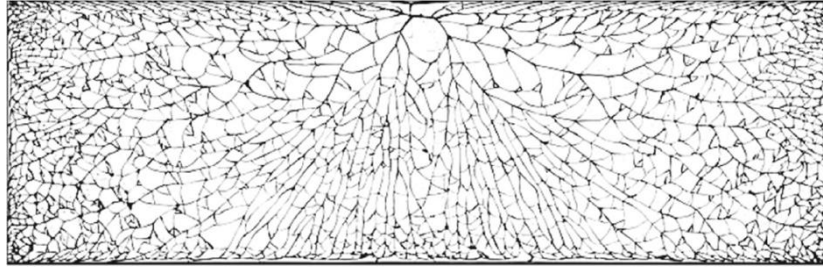
Types of fracture

Fractography



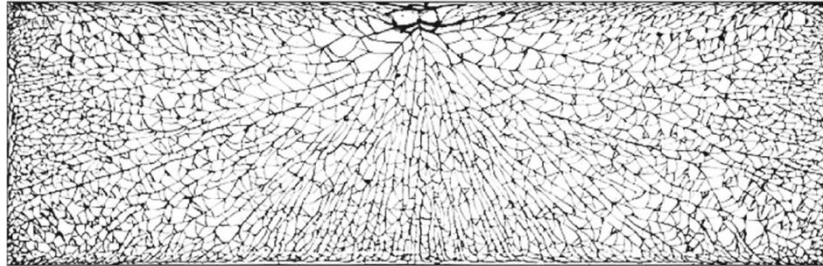
Types of fracture

Fractography



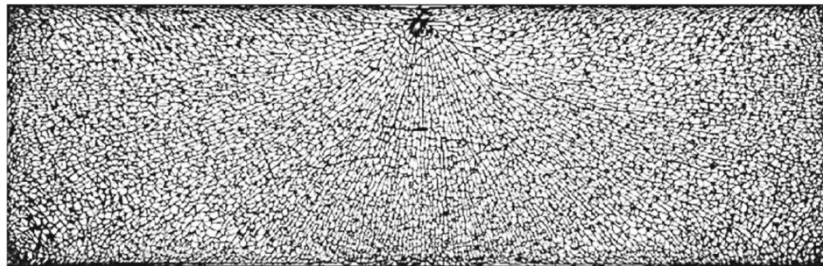
(a)

$$\sigma_m = 27 \text{ MPa}$$



(b)

$$\sigma_m = 30 \text{ MPa}$$



$$\sigma_m = 38 \text{ MPa}$$

Types of fracture

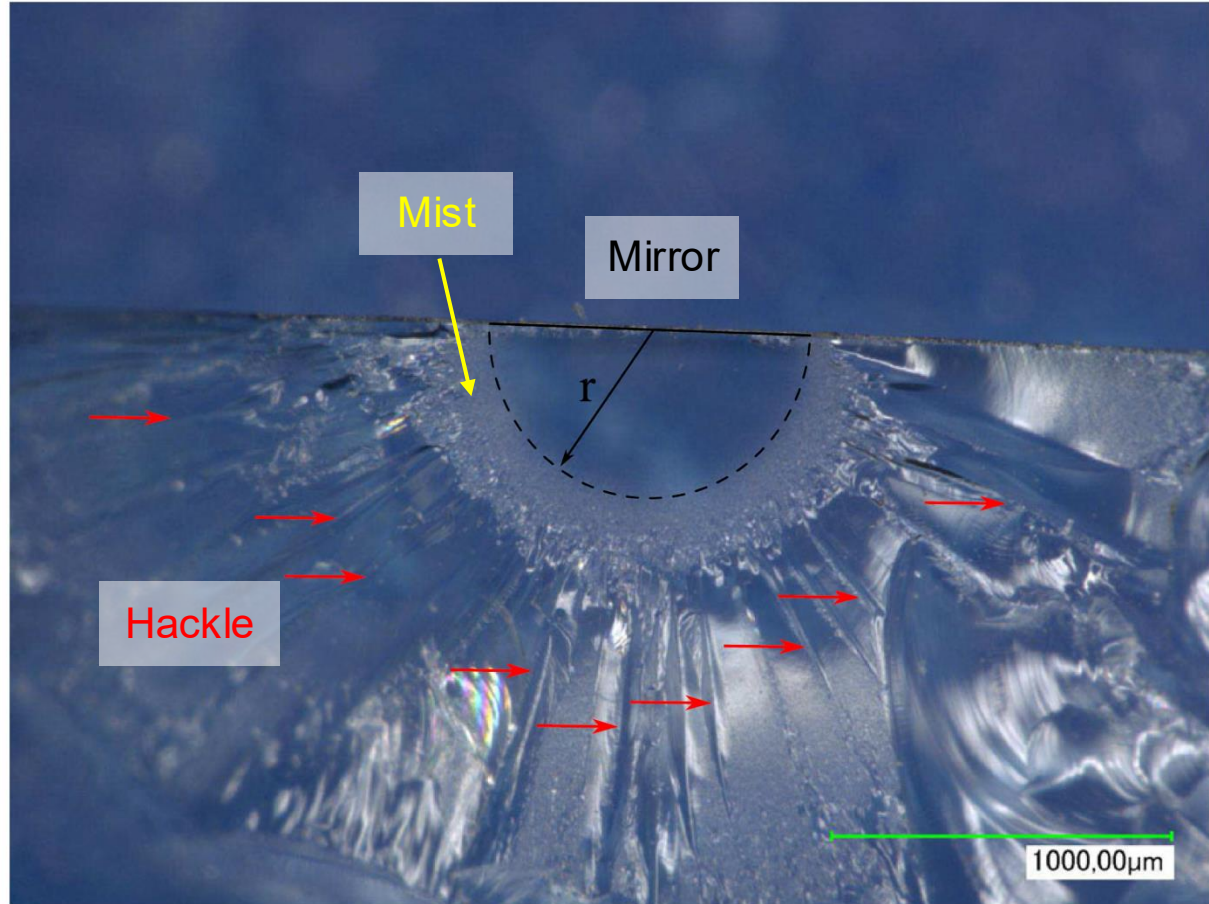
Fractography

Mirror

$$\sigma^{cr} = A_K r^{-1/2}$$

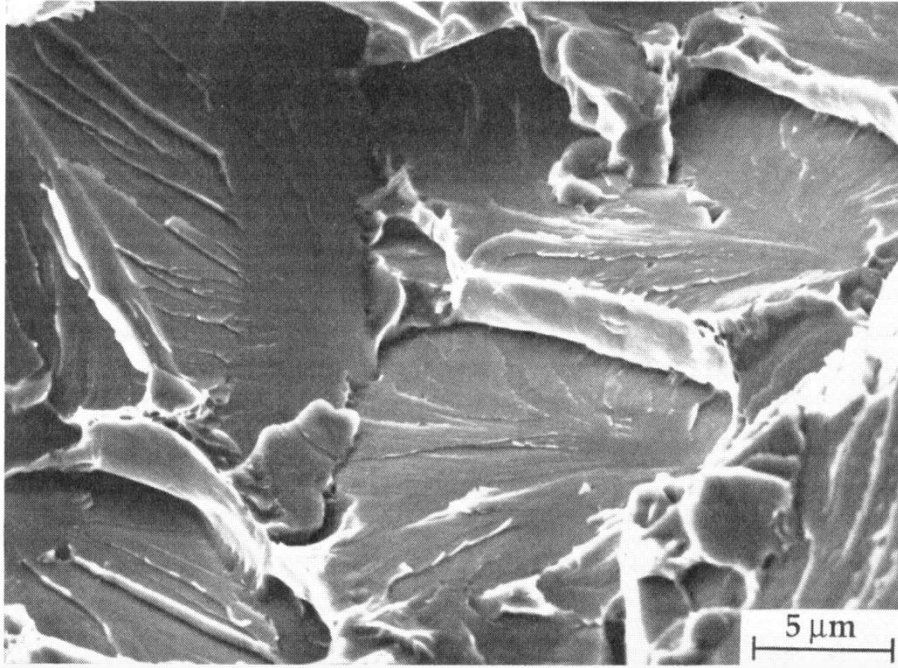


mirror
constant

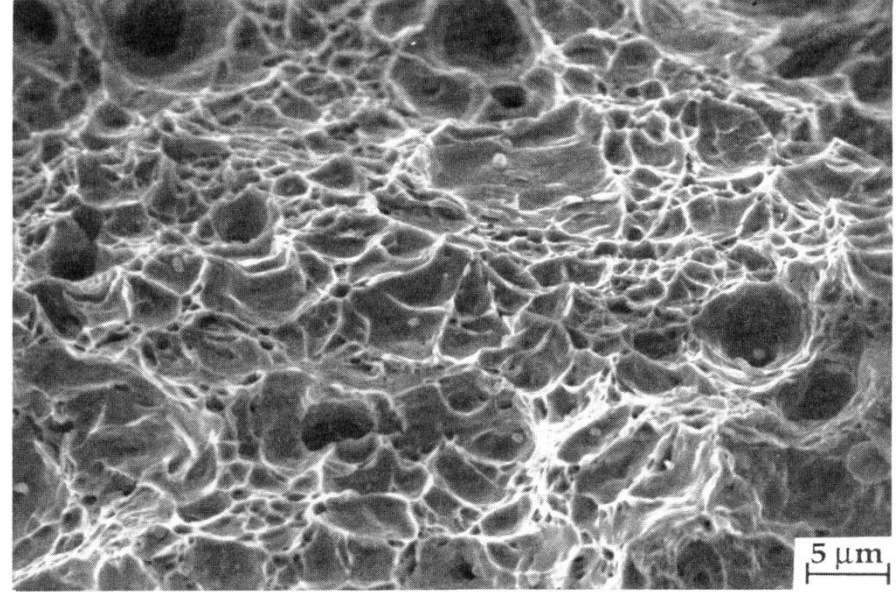


Types of fracture

Brittle / ductile



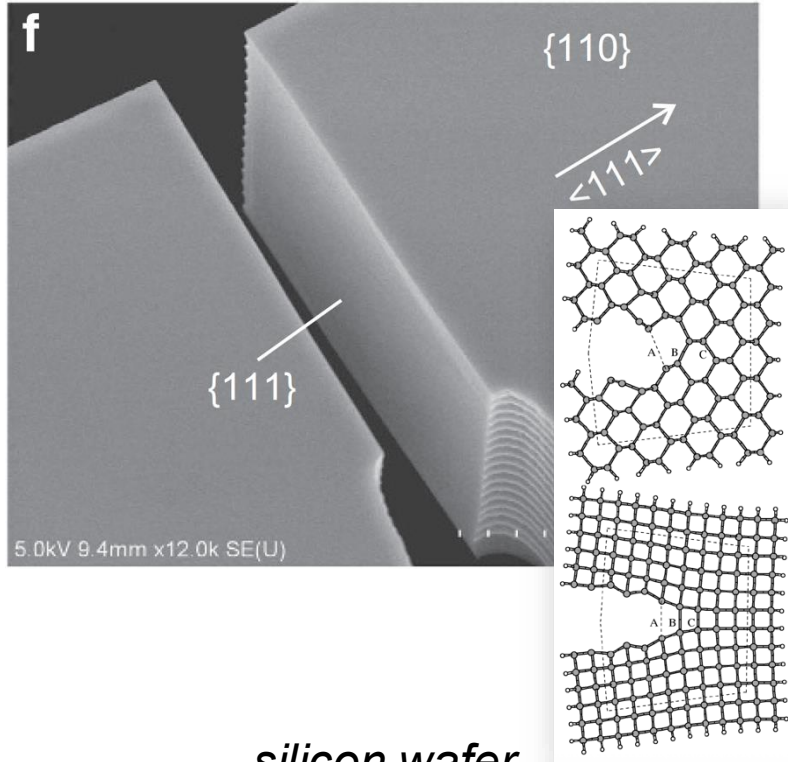
Brittle



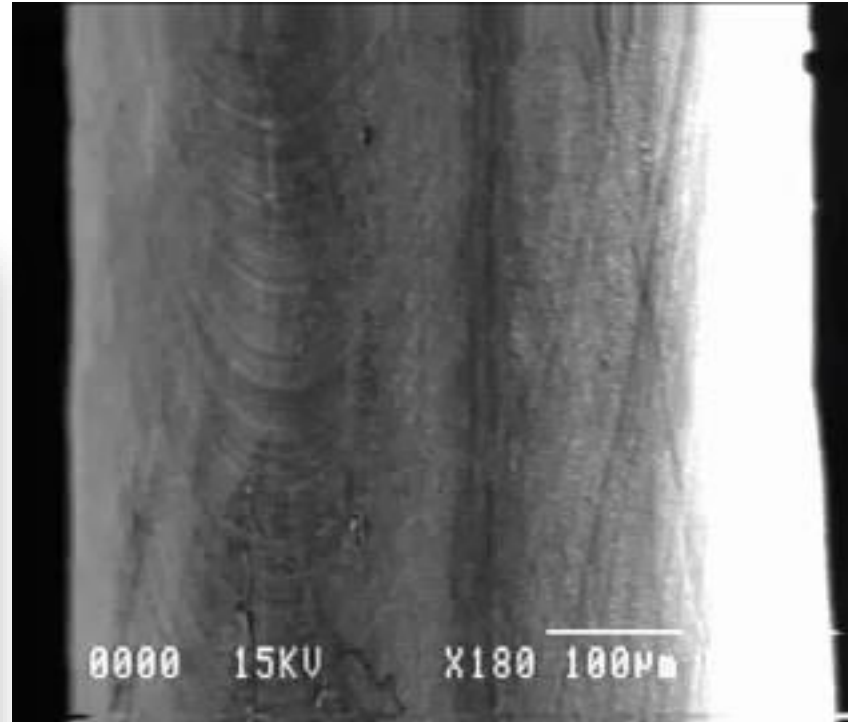
Ductile

Microscopic origin

Brittle / ductile



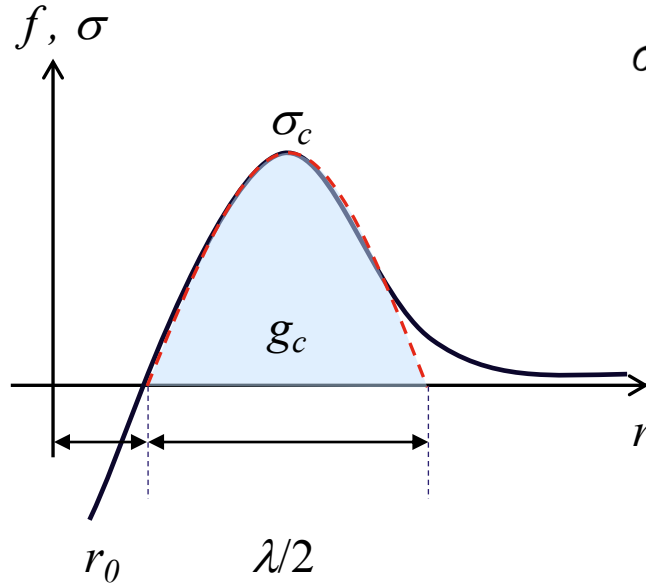
silicon wafer



single cadmium crystal

Microscopic origin

Relationship between strength and toughness



$$\sigma(r) = \sigma_c \sin\left(\frac{2\pi r}{\lambda}\right) \approx \sigma_c \frac{2\pi r}{\lambda}$$

$$\sigma = E \frac{r}{r_0} \quad \text{Hooke's law}$$

$$\sigma_c = \frac{E\lambda}{2\pi r_0}$$

$$\int_0^{\lambda/2} \sigma dr = -\sigma_c \frac{\lambda}{2\pi} \left[\cos\left(\frac{2\pi}{\lambda} \frac{\lambda}{2}\right) - \cos 0 \right] = \sigma_c \frac{\lambda}{\pi} = g_c$$

$$\sigma_c = \sqrt{\frac{Eg_c}{2r_0}}$$

$$E = 1050 \text{ GPa}$$

$$g_c = 5 - 20 \text{ J/m}^2$$

$$r_0 = 1.54 \cdot 10^{-10} \text{ m}$$

$$\sigma_c = 130 - 260 \text{ GPa}$$

$$(\sigma_{\text{exp}} = 200 - 250 \text{ GPa})$$



diamond

Types of fracture

Influencing factors

How many different ways are there to fracture a banana?

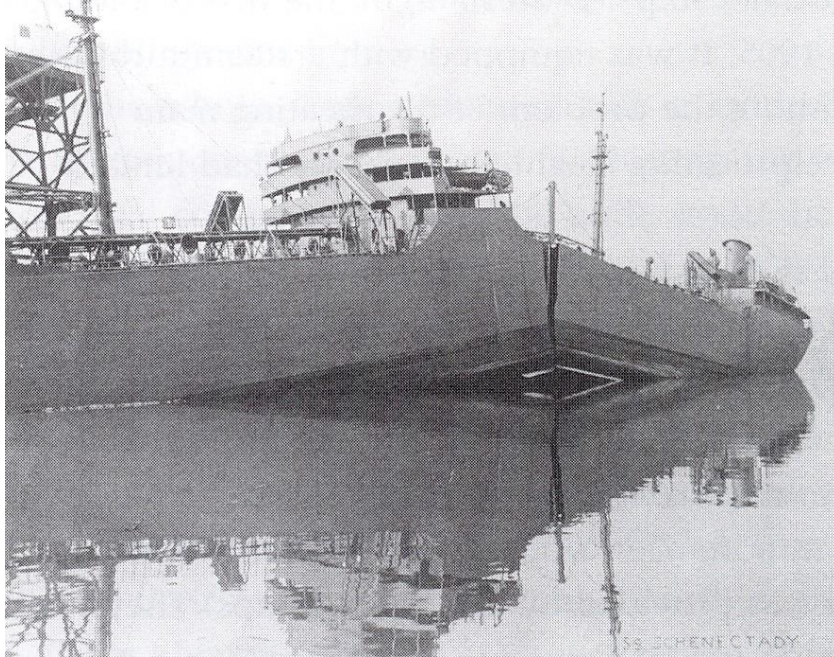
Taylor Sparks

https://www.youtube.com/watch?v=g_yOsz1lQBk

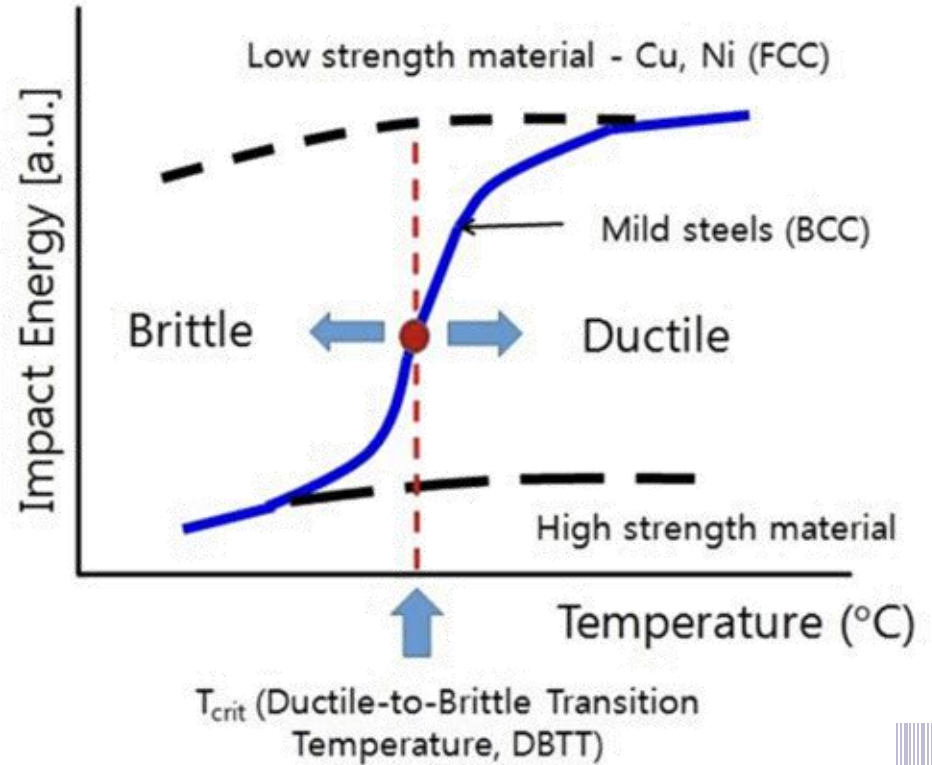


Types of fracture

Temperature



SS Schenectady, 1943



Types of fracture

Strain rate



Brittle



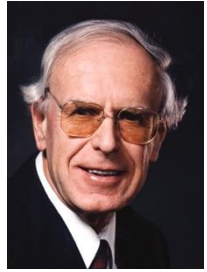
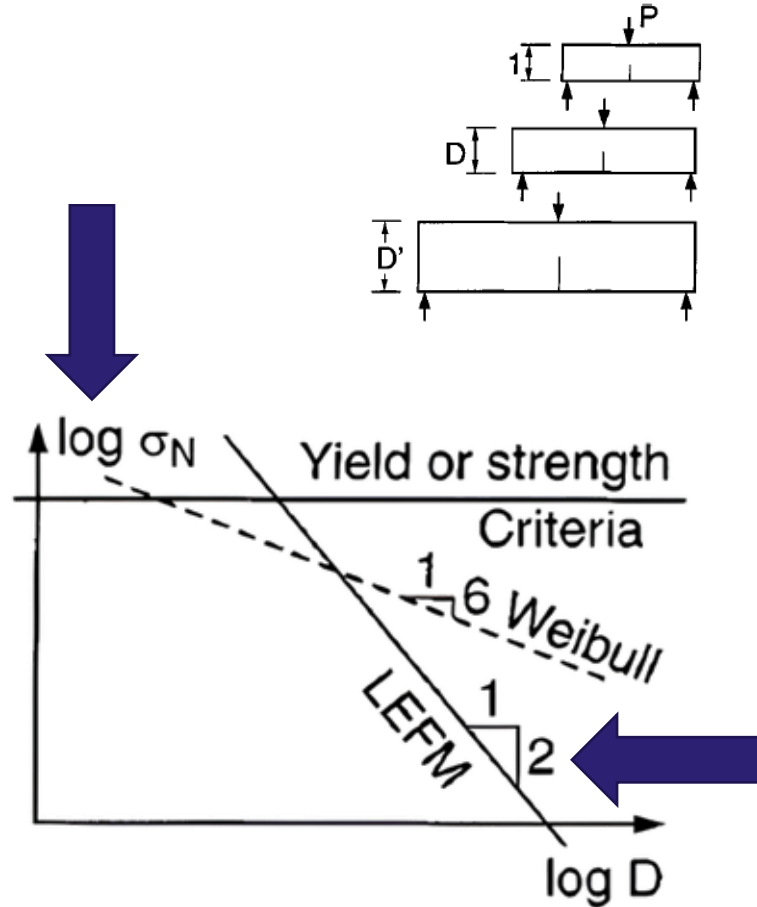
Ductile

Types of fracture

Size effect



Laboratory testing



Bažant et al. (1999)



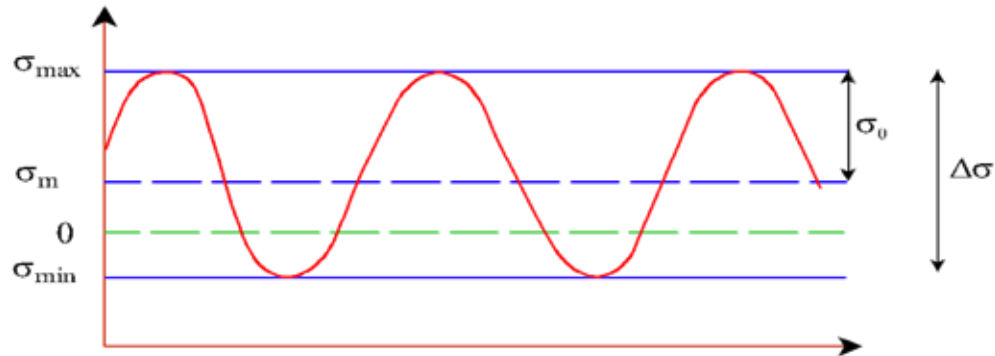
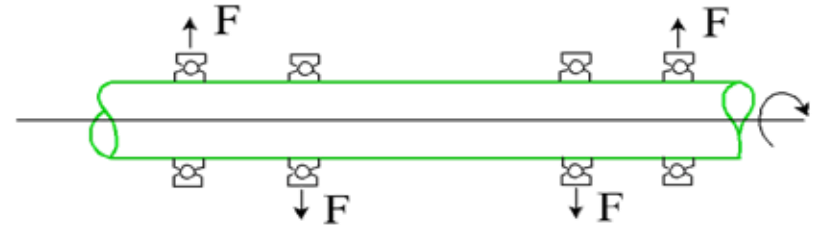
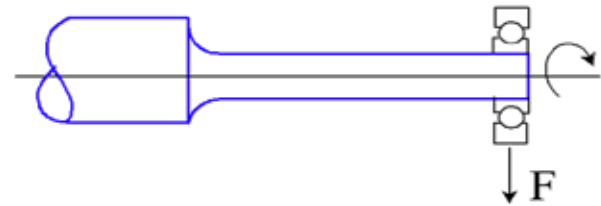
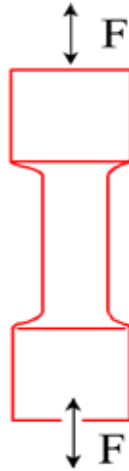
Hoover dam



Types of fracture

Repetition / fatigue

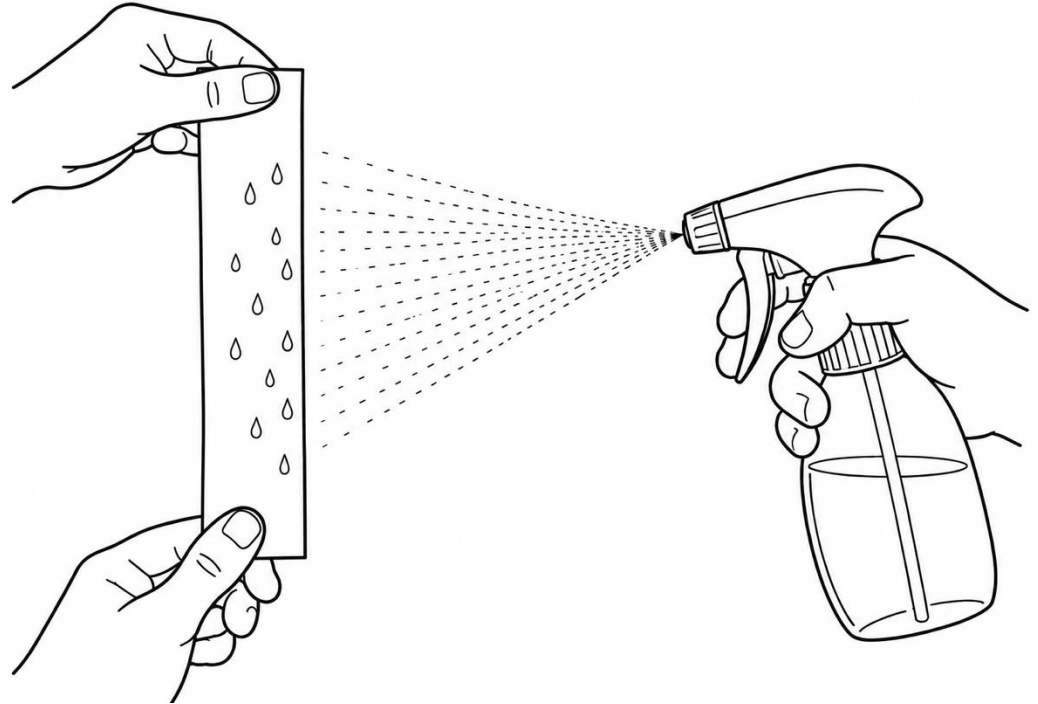
static repeated
 $K_{IC} > K_{IC}^{fatigue}$



Types of fracture

Environment

- Water / humidity
- Chemical environment
- Hydrogen
- Oxidizing/reducing atmosphere
- Irradiation
- Corrosive liquids / molten materials
- Thermal shock

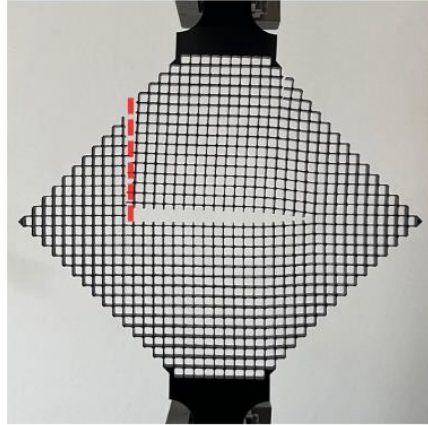


Types of fracture

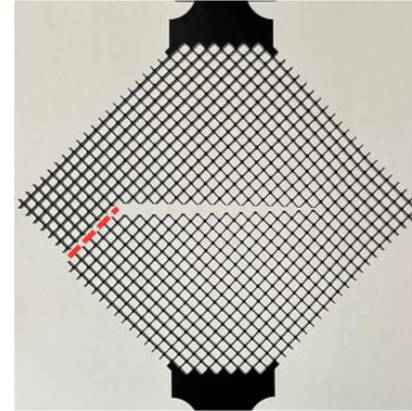
(Molnár & Réthoré, 2026)

Anisotropy

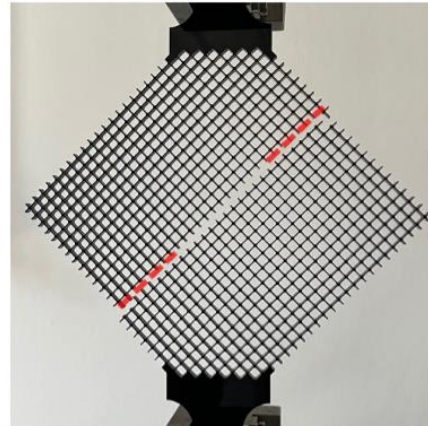
Tension



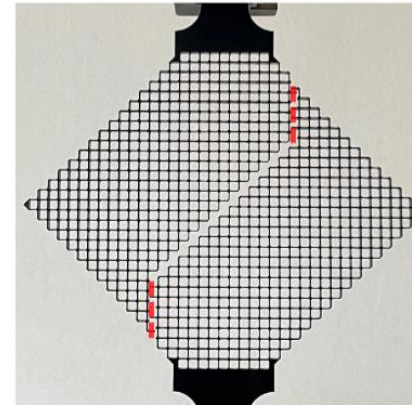
Case 4.



Shear

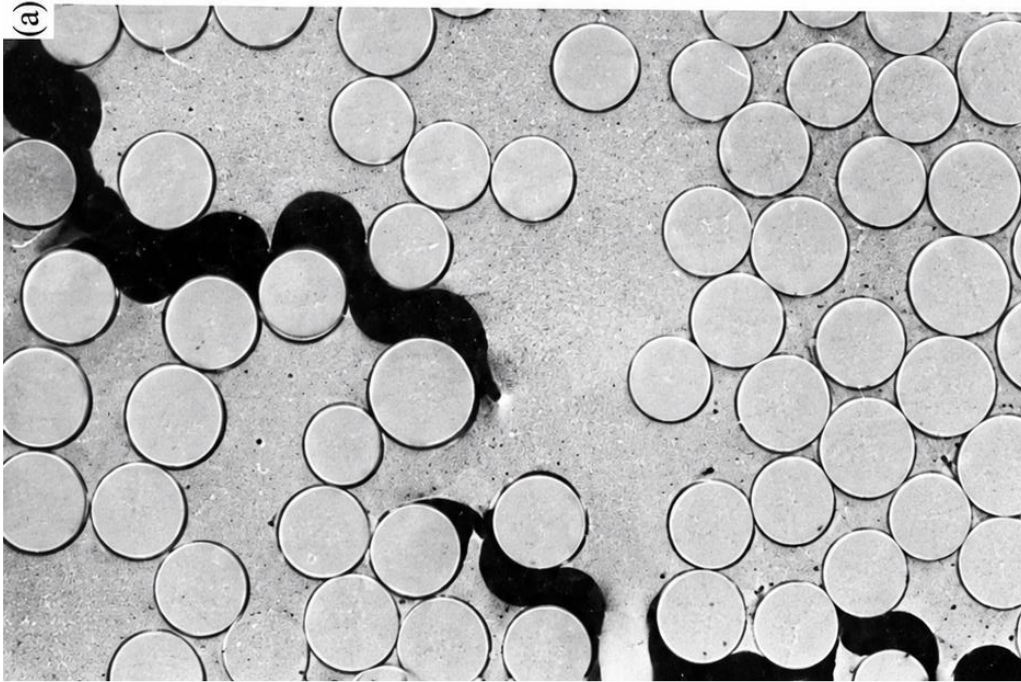


Case 3.

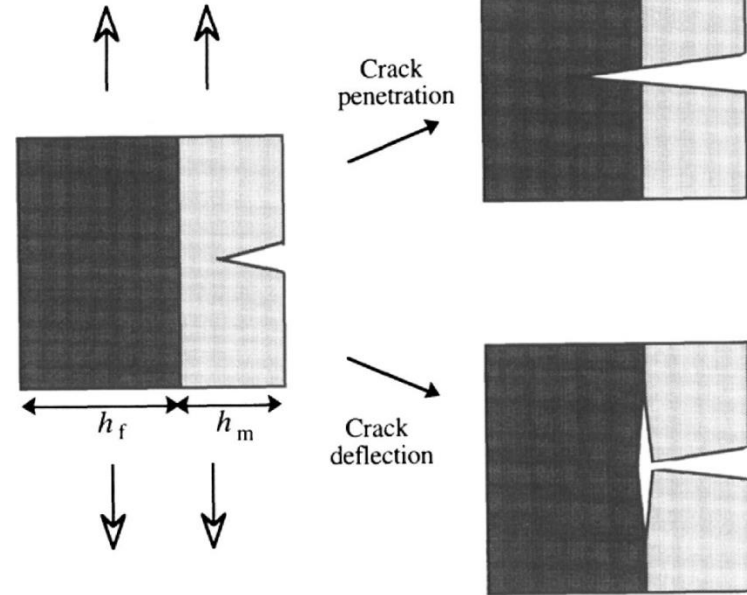


Types of fracture

Inhomogeneity



Hull (1992)



Summary

1. Fracture is a multi-physics problem
2. Fracture is a multi-scale problem
 - Spatially
 - Temporally
3. Instead of stress we use energy



Temperature
Loading rate
Environment
Fatigue
Size effect
Composition
Inhomogeneities
Anisotropy

References

1. Bažant, Z. P., Kim, J.-J. H., Daniel, I. M., Becq-Giraudon, E., & Zi, G. (1999). Size effect on compression strength of fiber composites failing by kink band propagation. **International Journal of Fracture, 95**, 103–141.
2. Bojtár, I. **Törésmechanika [Fracture Mechanics]**. Lecture notes, Budapest University of Technology and Economics, Department of Structural Mechanics, Budapest.
3. Broek, D. (1988). **The Practical Use of Fracture Mechanics**. Dordrecht: Kluwer Academic Publishers.
4. da Vinci, L. (c. 1500). **Codex Atlanticus**. Biblioteca Ambrosiana, Milan.
5. Galilei, G. (1638). **Discorsi e dimostrazioni matematiche, intorno à due nuove scienze**. Leiden: Elsevier.
6. Huang, P. Y., Kurasch, S., Srivastava, A., Skakalova, V., Kotakoski, J., Krashennnikov, A. V., Hovden, R., Mao, Q., Meyer, J. C., Smet, J., Muller, D. A., & Kaiser, U. (2012). Direct imaging of a two-dimensional silica glass on graphene. **Nano Letters, 12*(2)*, 1081–1086.
7. Mariotte, E. (1686). **Traité du mouvement des eaux et des autres corps fluides**. Paris: Estienne Michallet.
8. Molnár, G., & Réthoré, J. (2026). Fracture toughness of periodic beam lattices. **Journal of Theoretical, Computational and Applied Mechanics**, 1–24.
9. Reed, R. P., Smith, J. H., & Christ, B. W. (1983). **The Economic Effects of Fracture in the United States. Part 1: A Synopsis of the September 30, 1982 Report to the National Bureau of Standards by Battelle Columbus Laboratories**. NBS Special Publication 647-1. Washington, DC: U.S. Department of Commerce, National Bureau of Standards.
10. Duga, J. J., Fisher, W. H., Buxbaum, R. W., Rosenfield, A. R., Buhr, A. R., Honton, E. J., & McMillan, S. C. (1983). **The Economic Effects of Fracture in the United States. Part 2**. NBS Special Publication 647-2. Washington, DC: U.S. Department of Commerce, National Bureau of Standards.
11. Zachariasen, W. H. (1932). The atomic arrangement in glass. **Journal of the American Chemical Society, 54*(10)*, 3841–3851.