

Numerical modeling of brittle fracture using the phase-field method

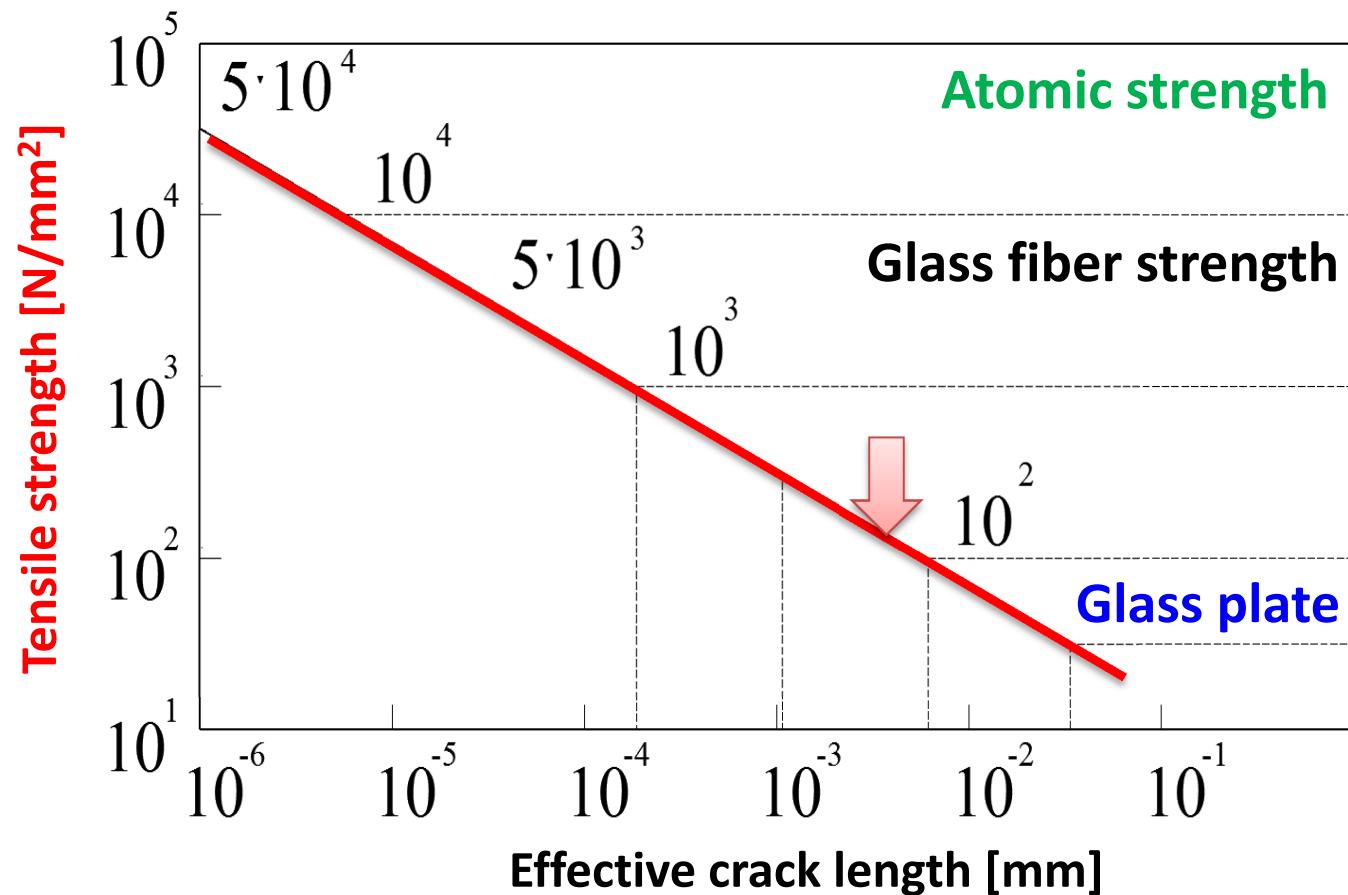
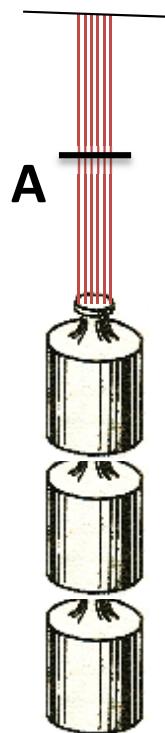
Gergely Molnár and Anthony Gravouil



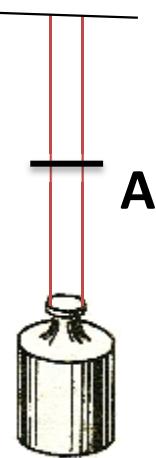
Introduction

Macroscopic strength

FIBERGLASS
TRUSS



GLASS
ROD

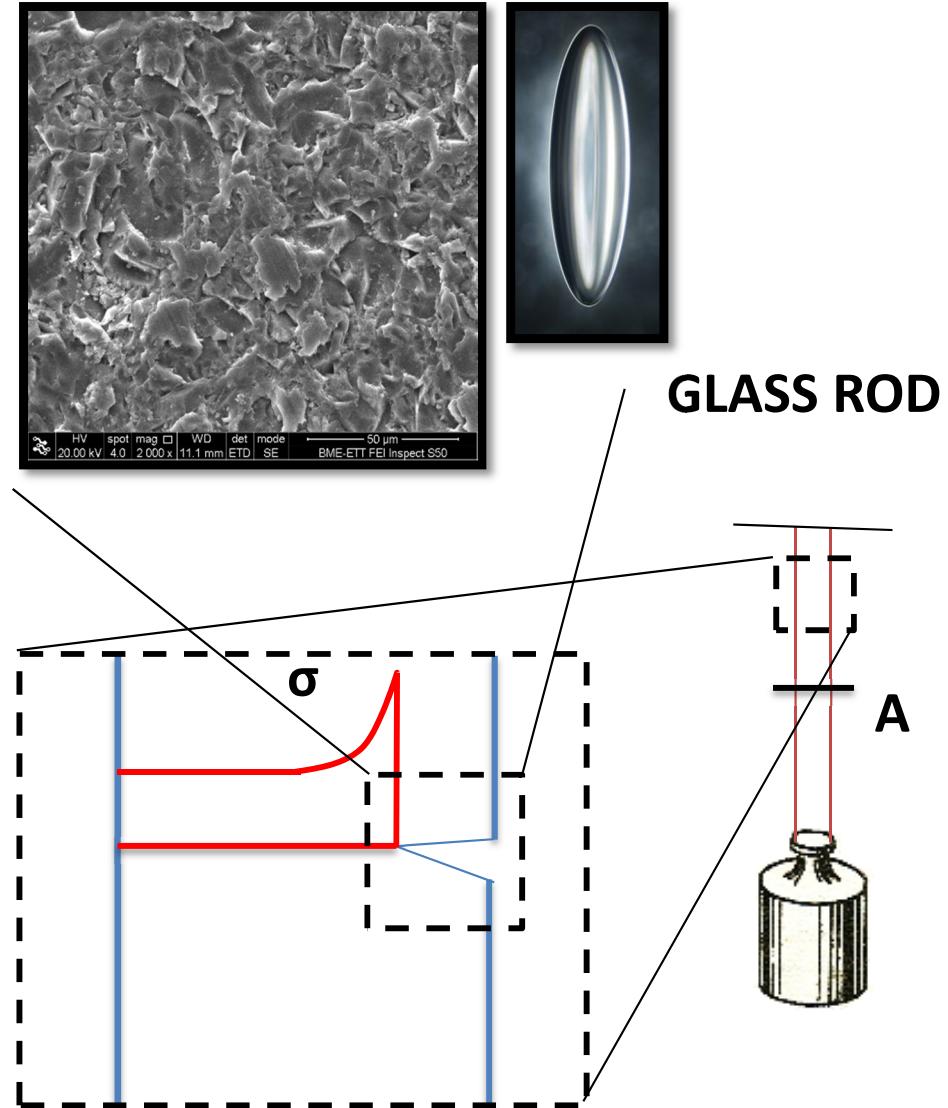
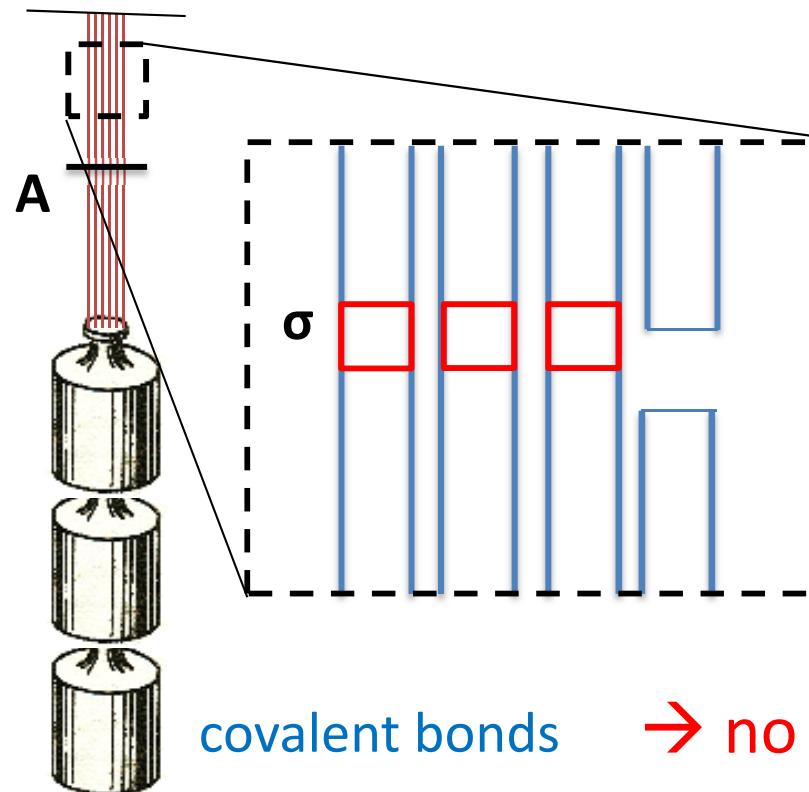


Wörner, 2001.

Introduction

Macroscopic strength

FIBERGLASS TRUSS

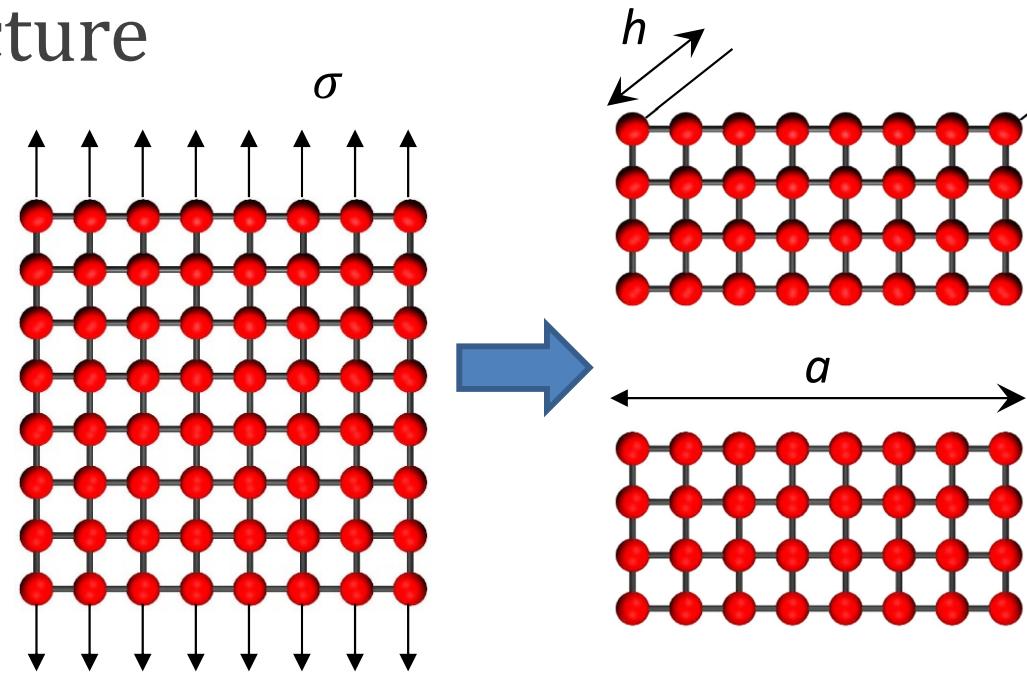
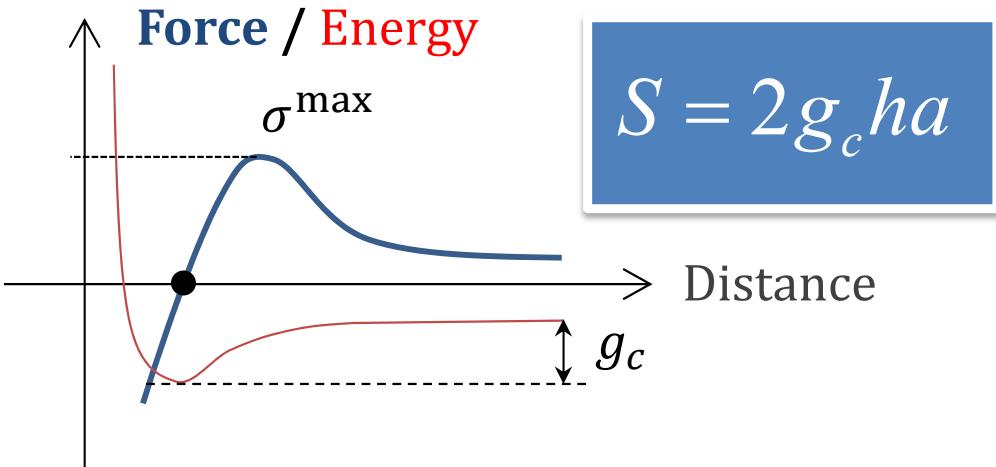


covalent bonds → no stress redistribution

→ brittle

Introduction

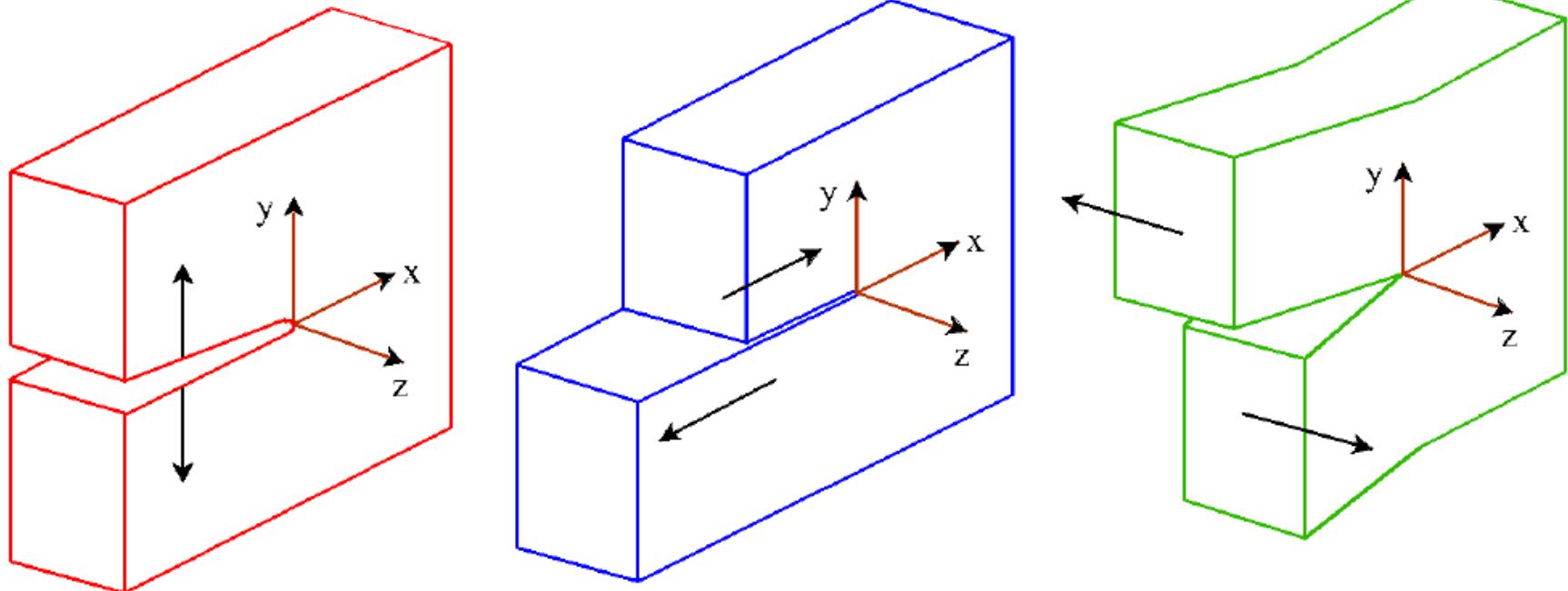
Griffith theory of brittle fracture



Crack modes

Irwin, 1957

Erdogan , 2000



Introduction

How do we approximate it with a phase-field?

1. Brittle fracture

$$-\frac{\partial \psi}{\partial a} = \frac{\partial S}{\partial a} = g_c$$

Griffith, 1920

2. Minimization problem

$$E(\mathbf{u}, \Gamma) = \int_{\Omega} \psi(\varepsilon(\mathbf{u})) d\Omega + g_c \int_{\Gamma} d\Gamma$$

Mumford & Shah, 1989

Francfort & Marigo, 1998

3. Crack energy density

$$E(\mathbf{u}, \mathbf{d}) = \int_{\Omega} g(\mathbf{d}) \psi(\varepsilon(\mathbf{u})) d\Omega + g_c \int_{\Omega} \left(\frac{1}{2l_c} \mathbf{d}^2 + \frac{l_c}{2} |\nabla \mathbf{d}|^2 \right) d\Omega$$

$l_c \rightarrow 0$ Γ converges

$\dot{d} \geq 0$ crack energy density - γ



Ambrosio & Tortorelli, 1990

Bourdin et al., 2000

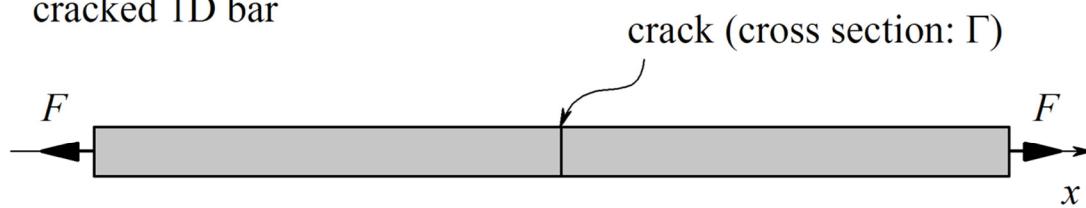
Amor et al., 2009

Miehe et al., 2010a

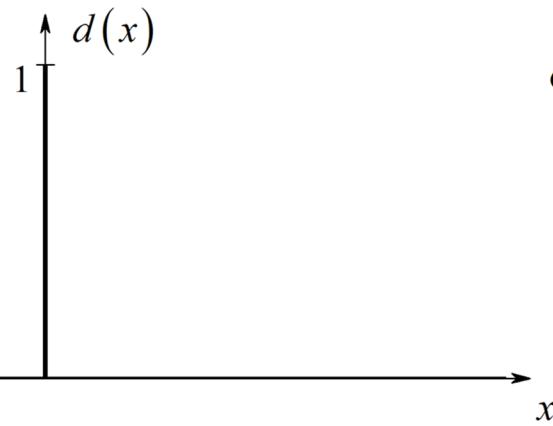
Introduction

What is diffuse damage?

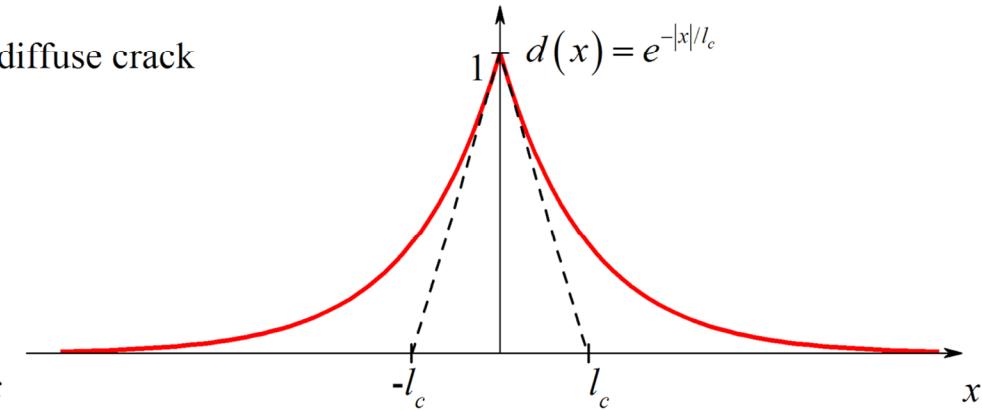
cracked 1D bar



sharp crack



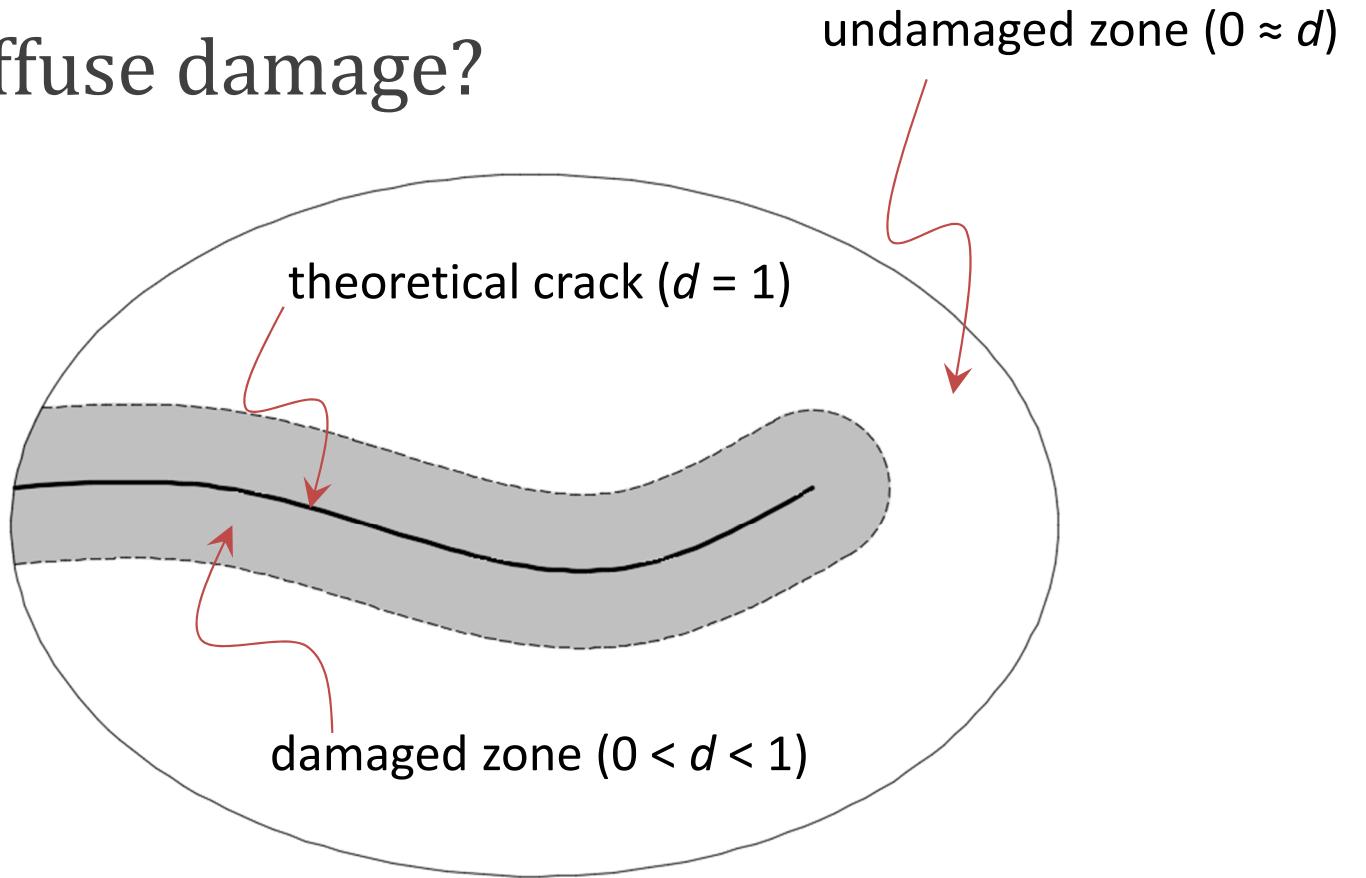
diffuse crack



Solving **fracture mechanics** problem with
Partial Differential Equations (**PDEs**)

Introduction

What is diffuse damage?



Solving **fracture mechanics** problem with
Partial Differential Equations (**PDEs**)

Prof. Dr.-Ing.
Christian Miehe
(1956 – †2016)



Phase-field method

Staggered scheme

Miehe et al., 2010b

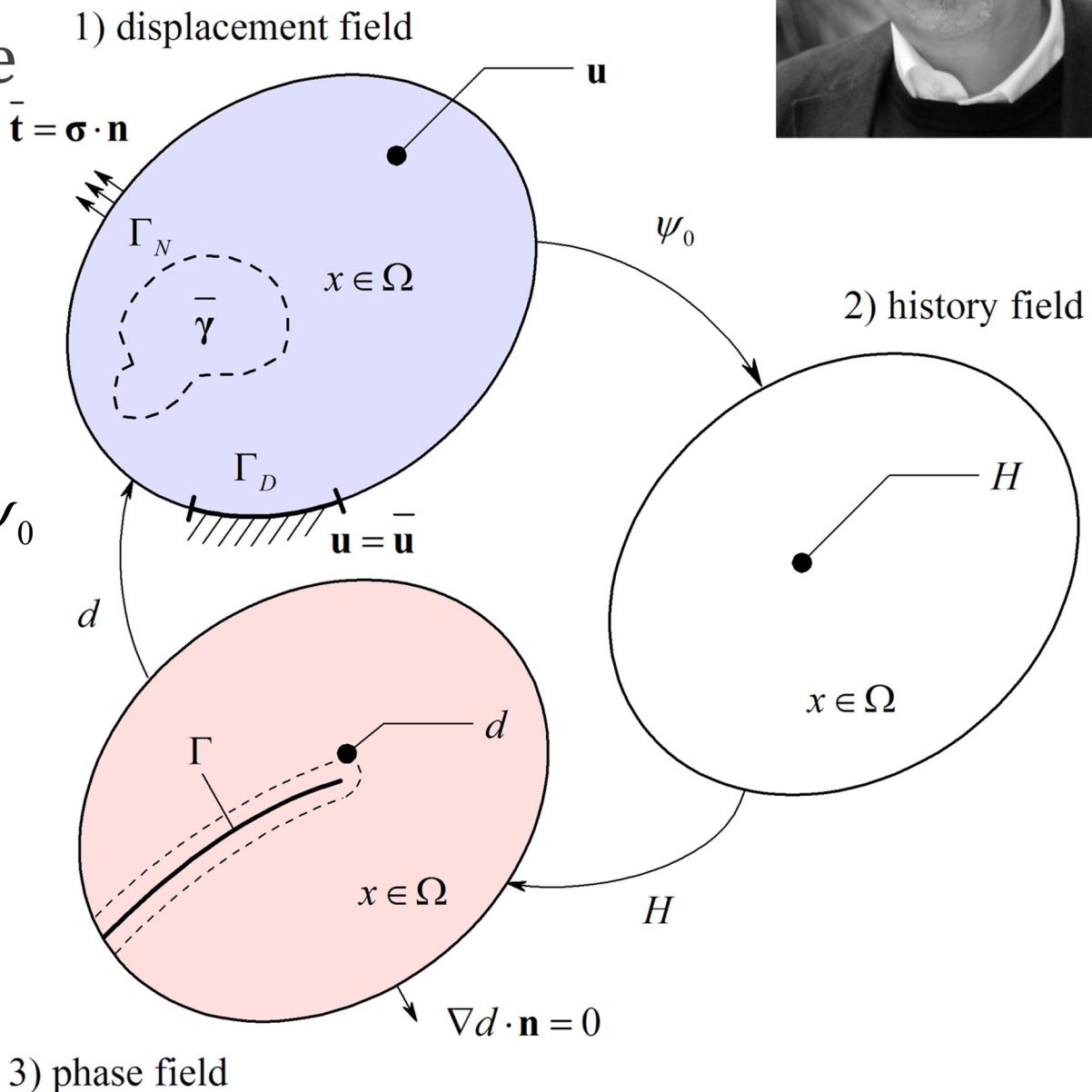
$$E^u(\mathbf{u}, d)$$

$$E^d(d, H_n)$$

$$H_n = \psi_0(\varepsilon(\mathbf{u})) \quad \text{if} \quad H_{n-1} < \psi_0$$

Robustness!!!

Efficiency?



Phase-field method

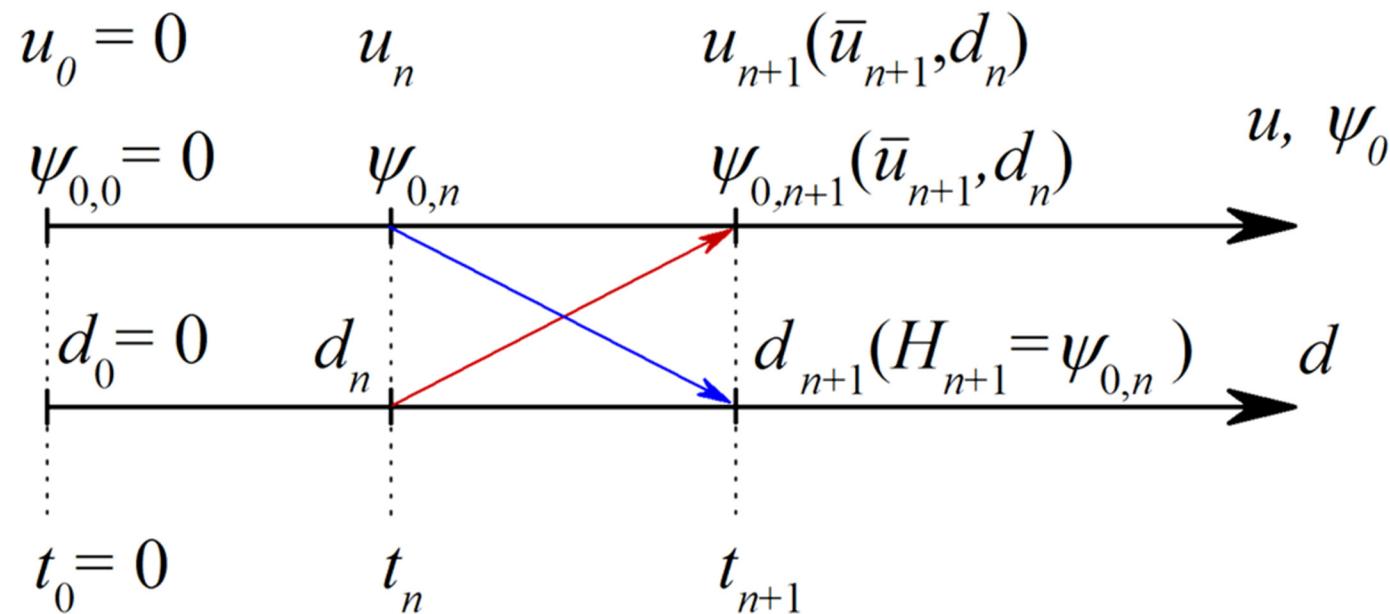
Staggered scheme



Displacement field

Phase-field

Time



$$E^u(\mathbf{u}, d)$$

$$E^d(d, H_n)$$

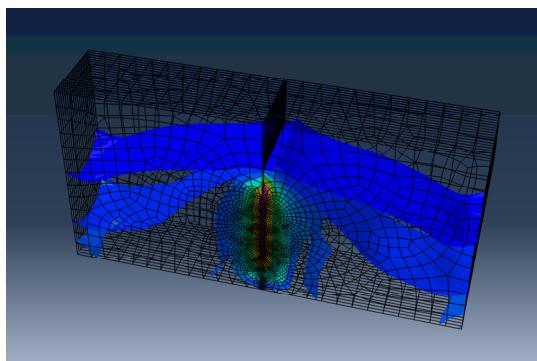
$$H_n = \psi_0(\varepsilon(\mathbf{u})) \quad \text{if } H_{n-1} < \psi_0$$

Open source implementation

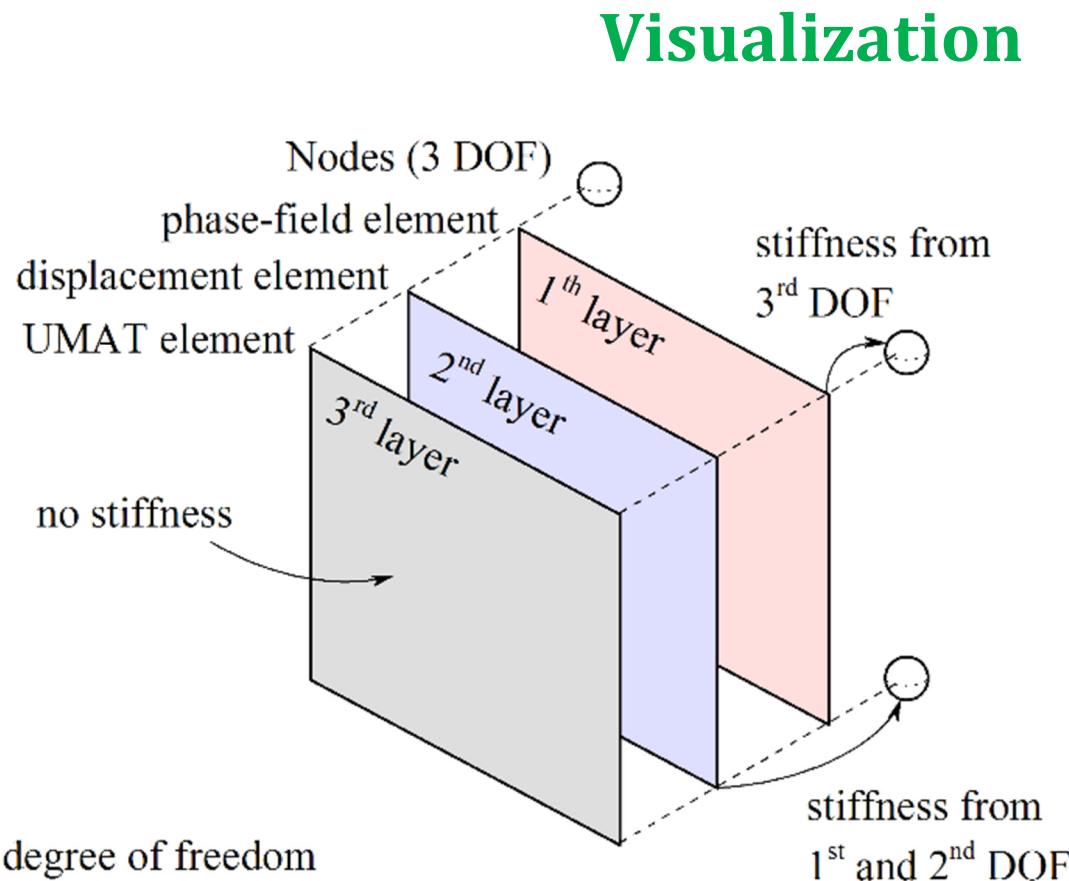
ABAQUS/UEL option (ABAQUS + FORTRAN compiler)

stiffness matrix + residue vector for every element

FORTRAN and ABAQUS
files are available in
both **2D** and **3D**

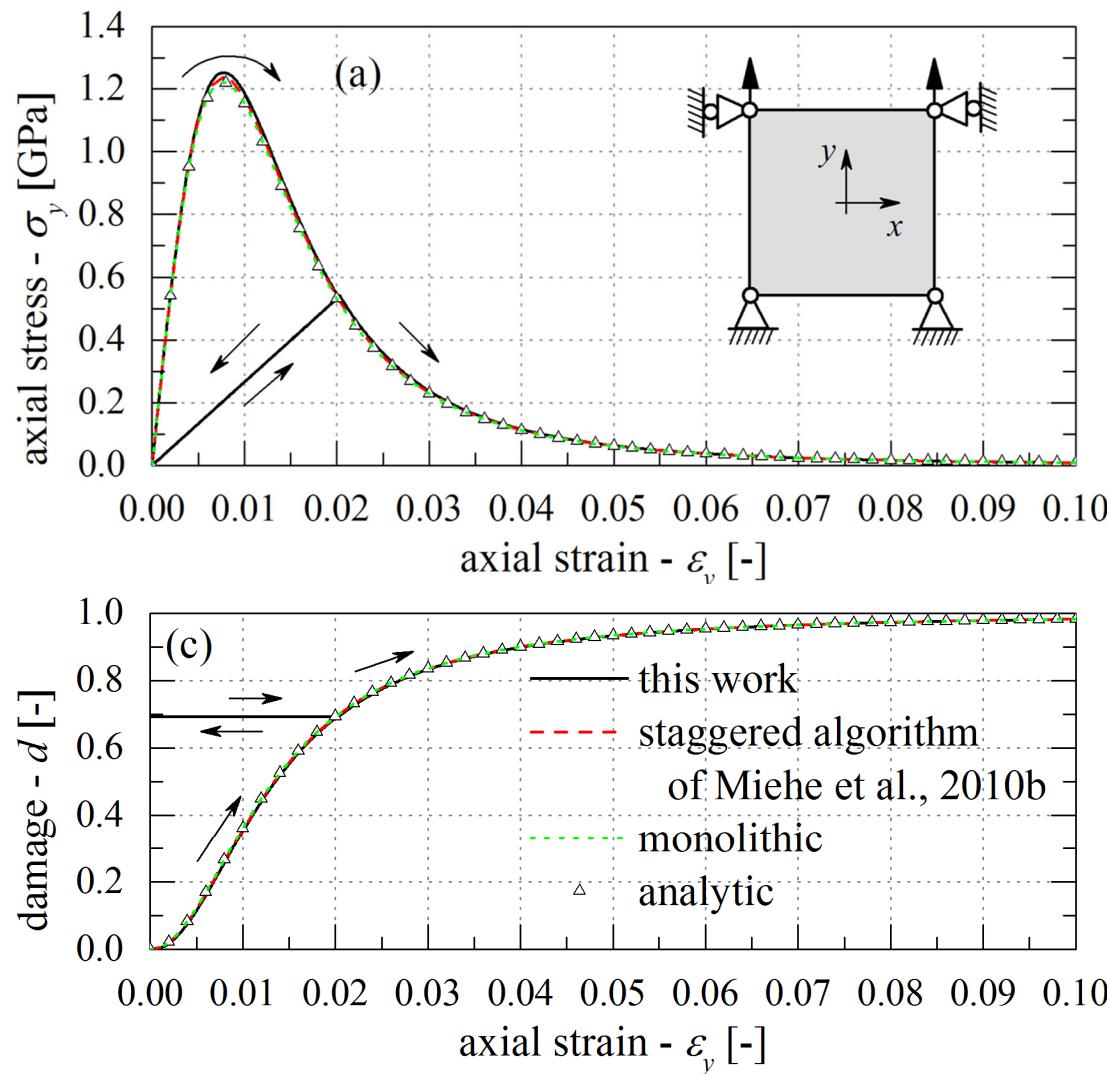


DOF - degree of freedom



Phase-field method c_{22} - (2,2) element of the stiffness matrix

Single element solution

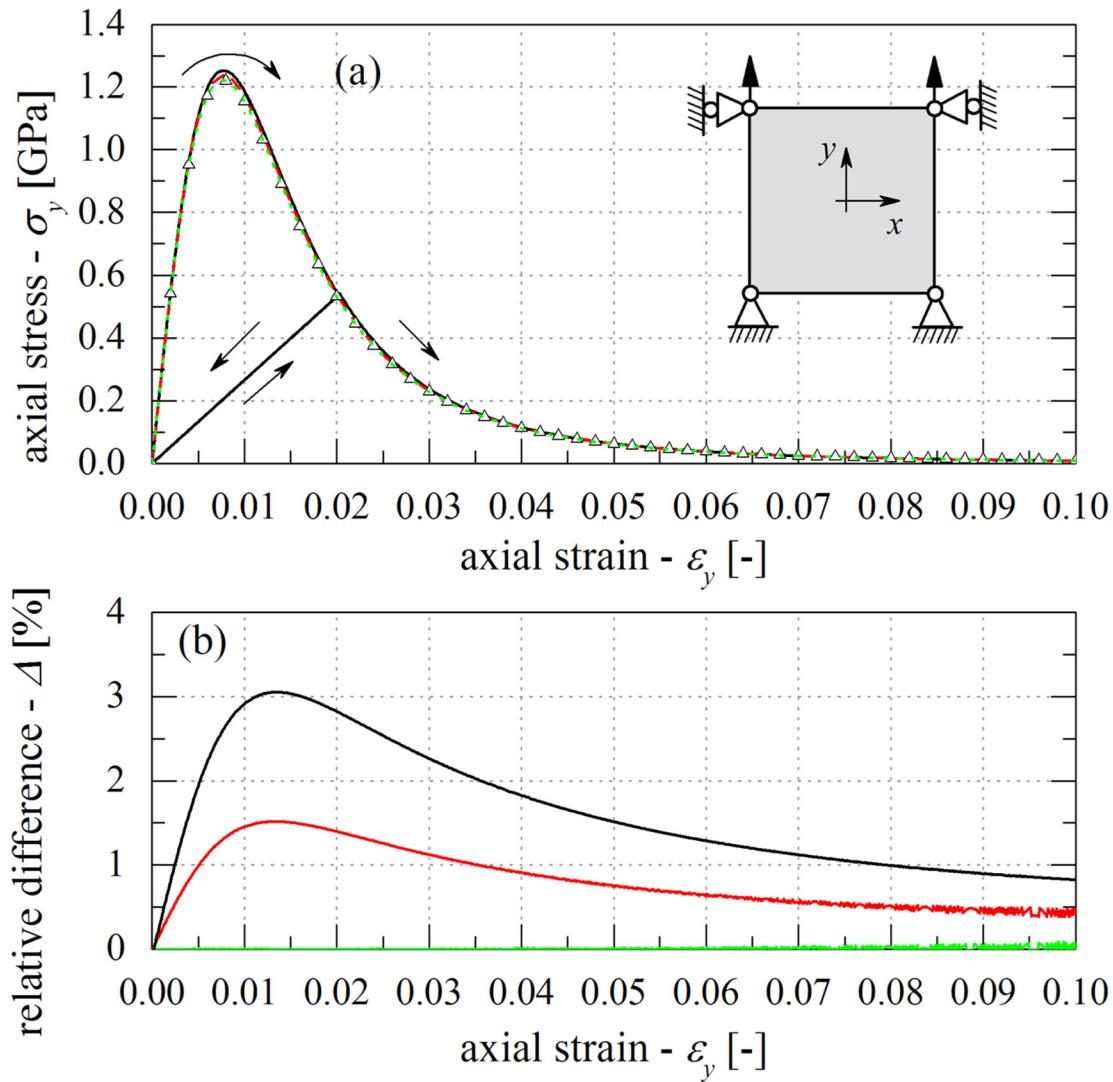


$$\sigma_y = (1-d)^2 \varepsilon_y c_{22}$$

$$d = \frac{\varepsilon_y^2 c_{22}}{\frac{g_c}{l_c} + \varepsilon_y^2 c_{22}}$$

Phase-field method

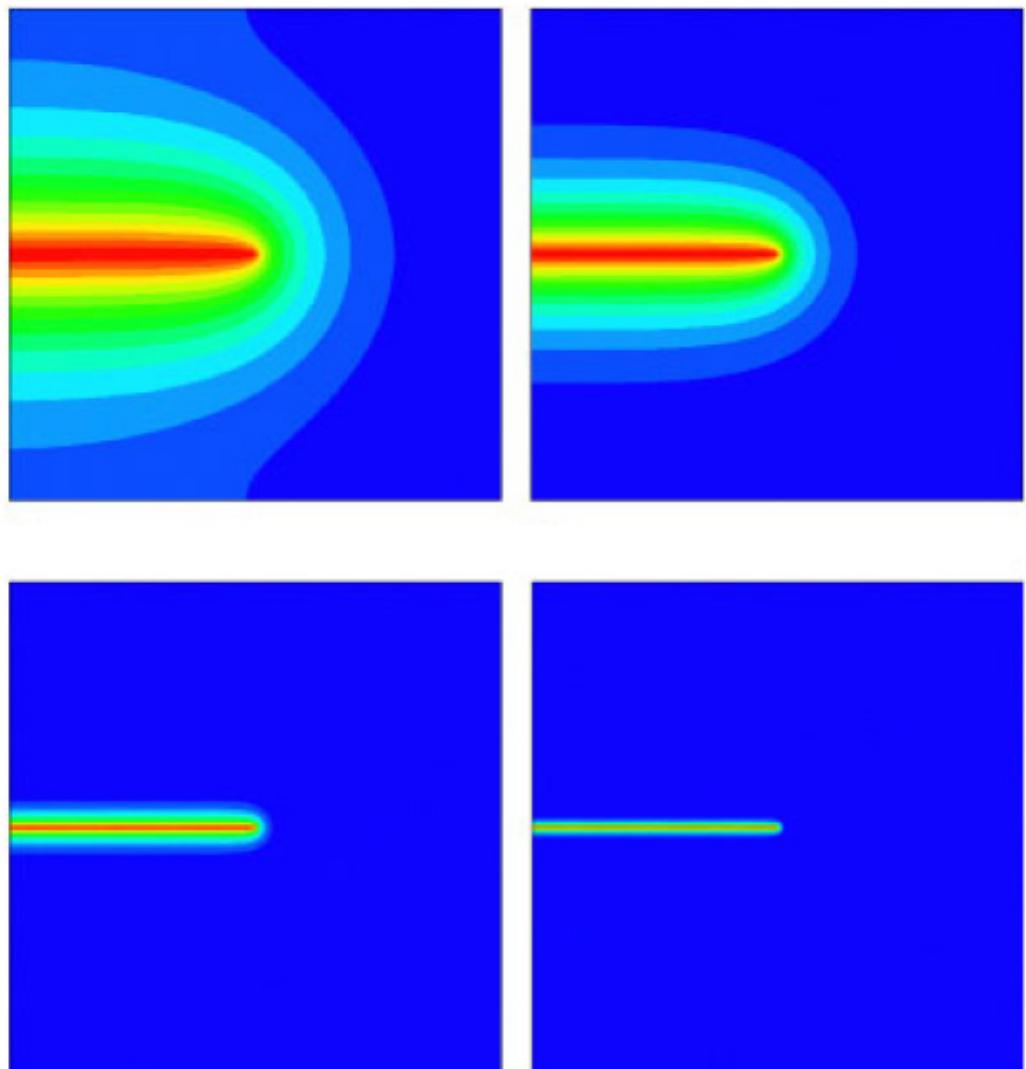
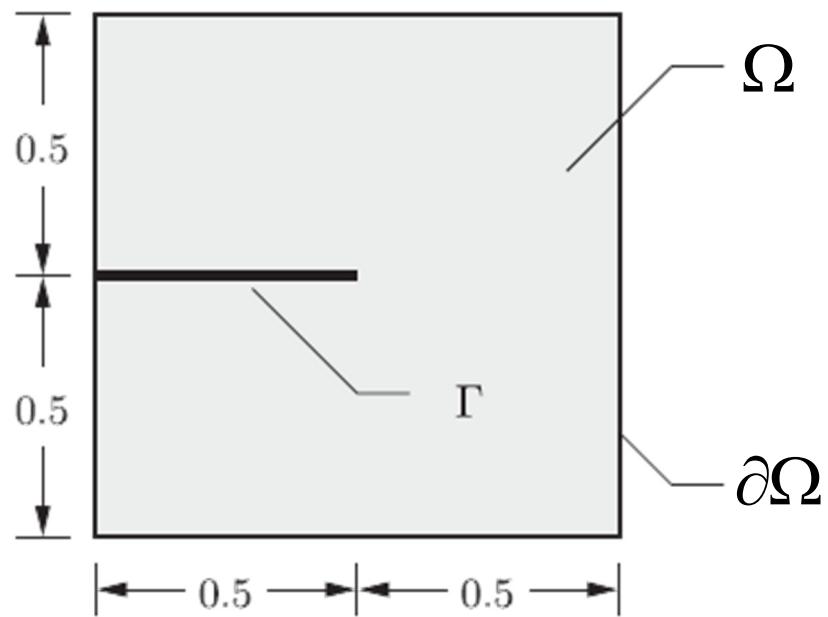
Single element solution



$$\Delta = \frac{\sigma_y - \sigma_y^{\text{analytic}}}{\sigma_y^{\text{analytic}}}$$

Parameters

How fine should the mesh be?

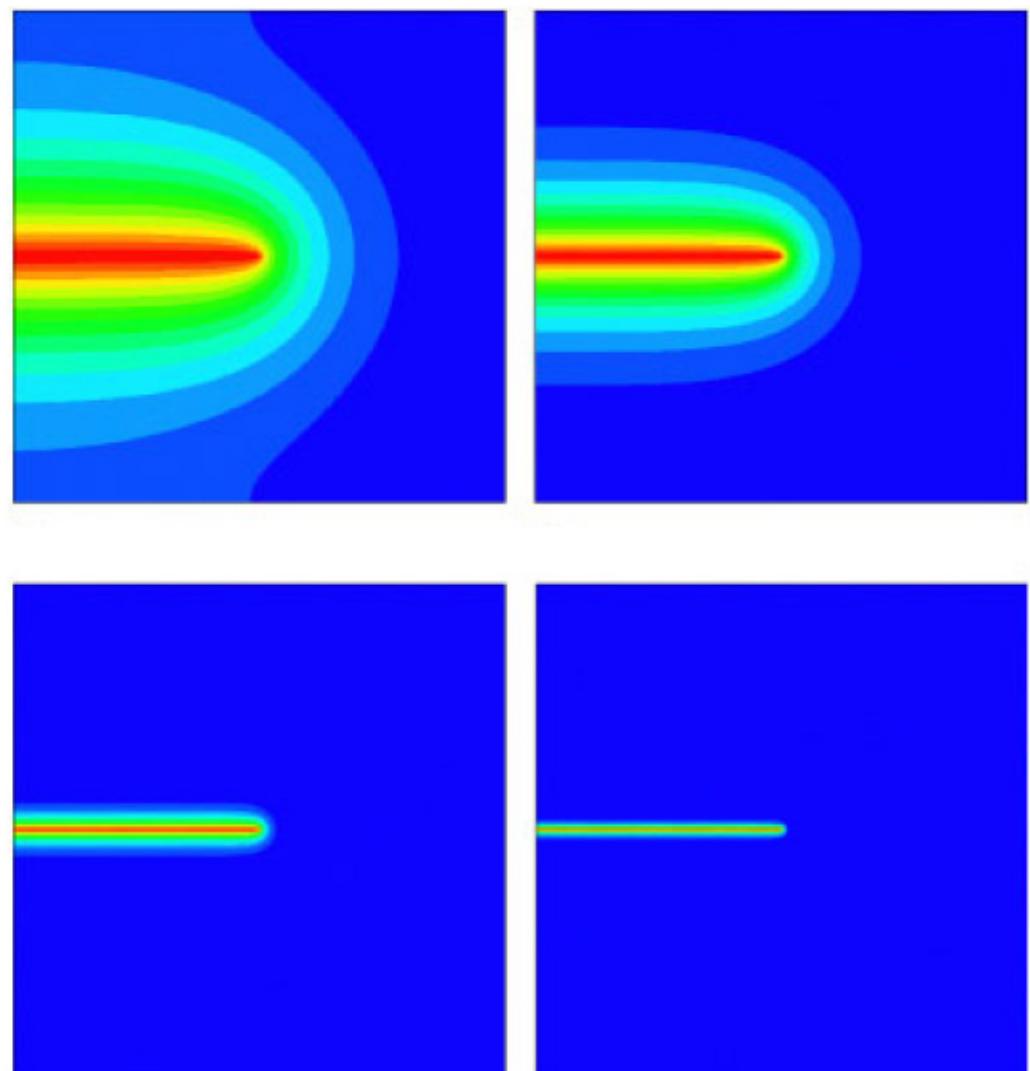
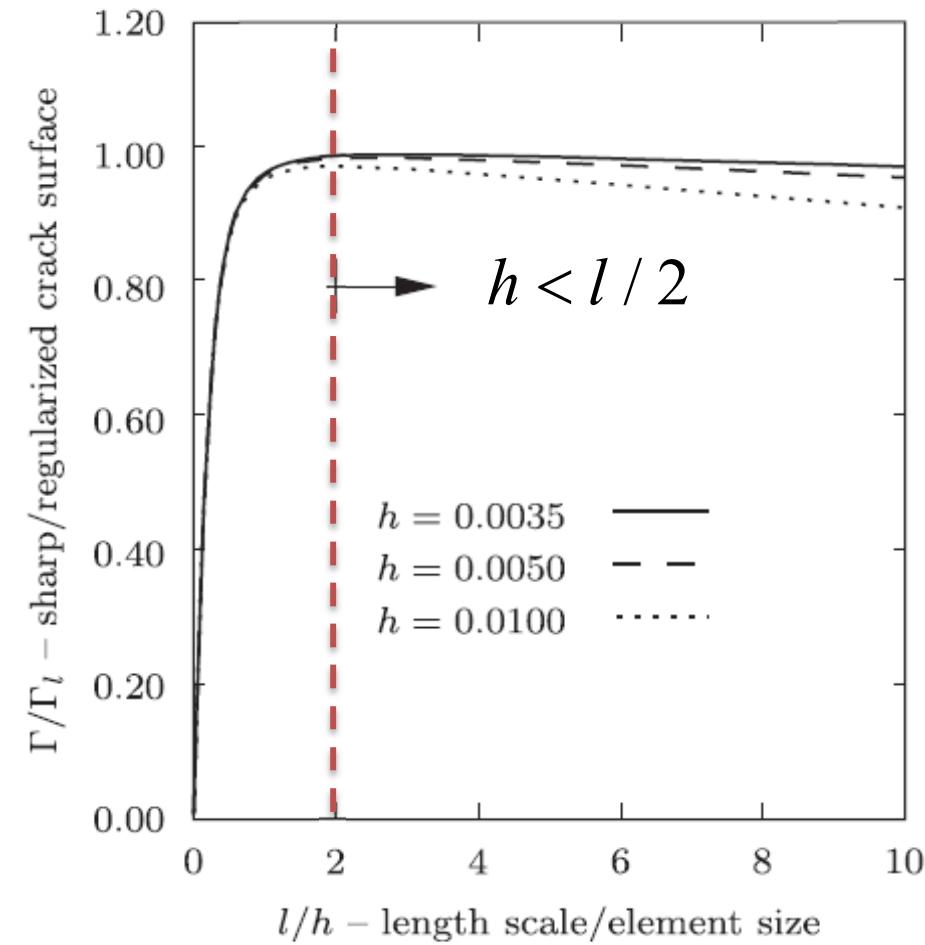


Parameters

How fine should the mesh be?

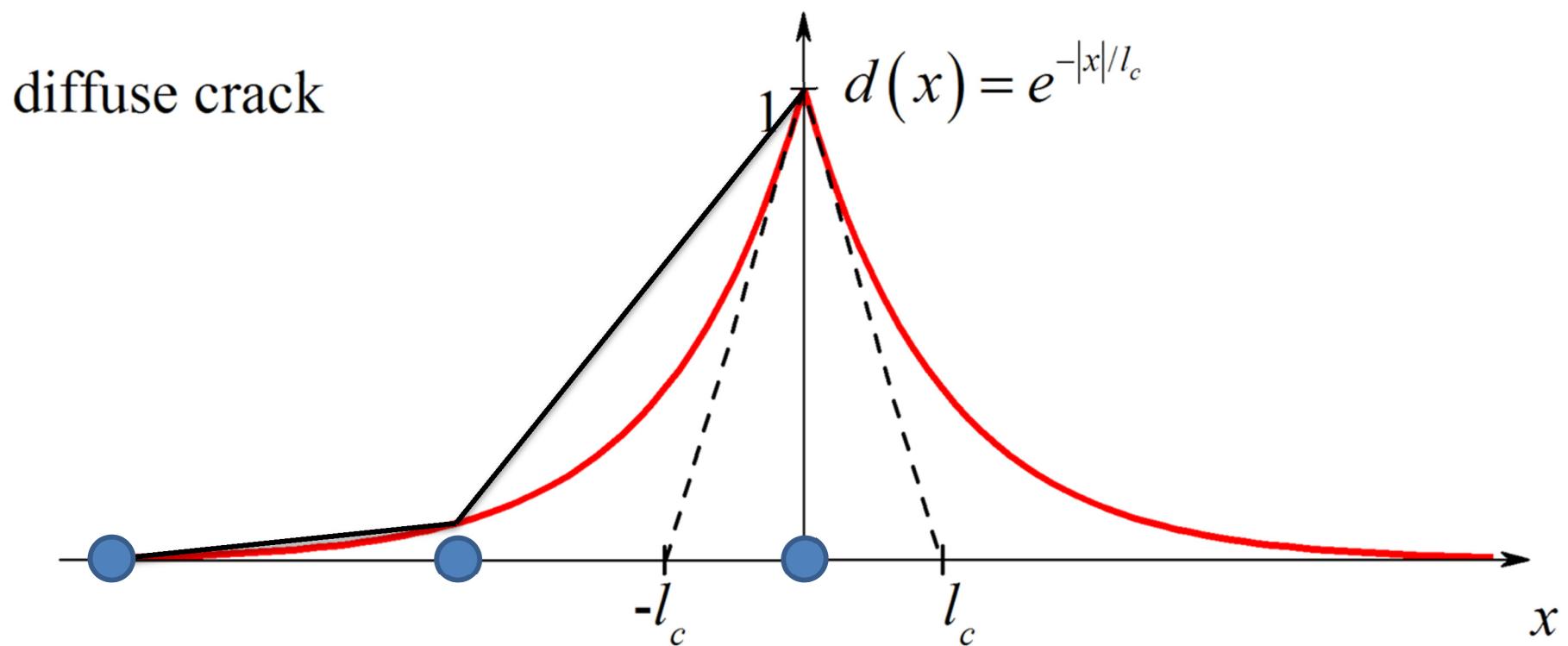
theoretical $\longrightarrow \Gamma = 0.5$

$$\Gamma_l = \int_{\Omega} \gamma d\Omega$$



Parameters

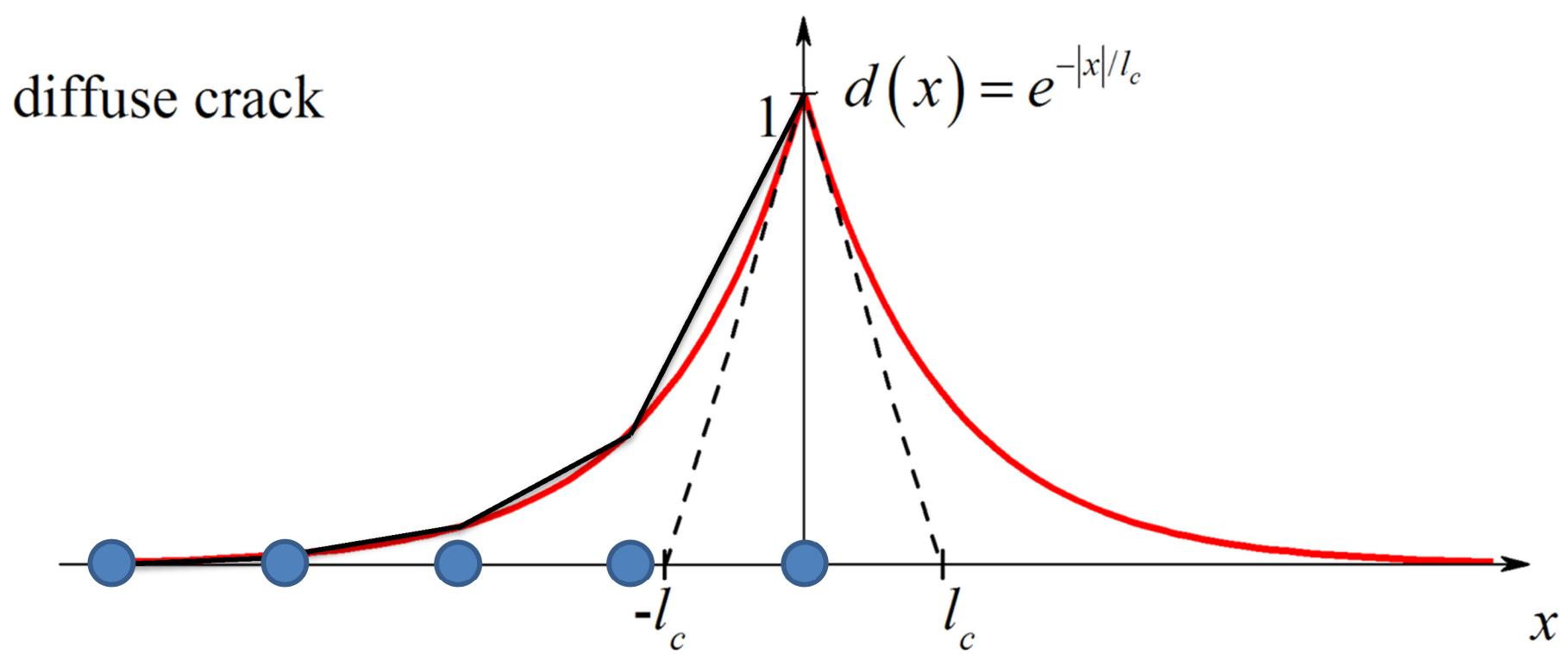
How fine should the mesh be?



Solving **fracture mechanics** problem with
Partial Differential Equations (**PDEs**)

Parameters

How fine should the mesh be?

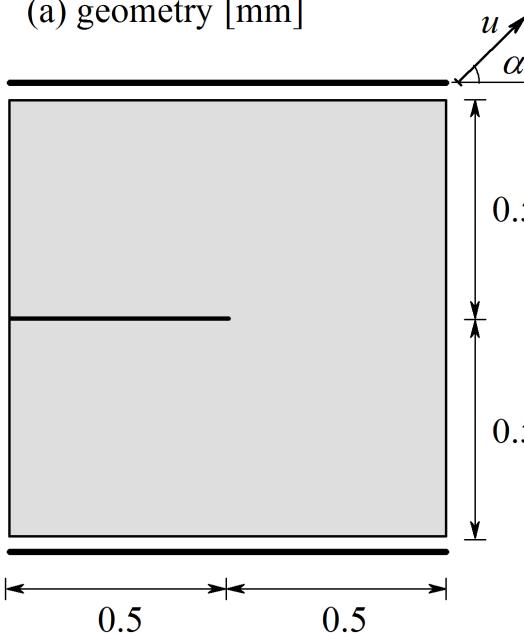
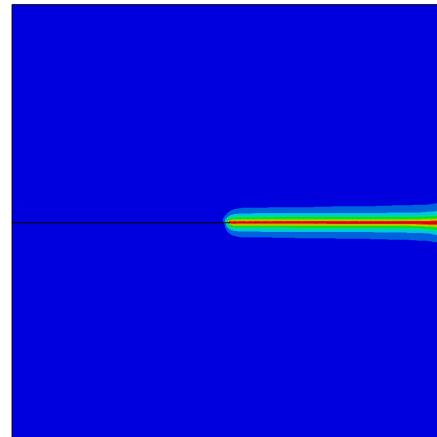


Solving **fracture mechanics** problem with
Partial Differential Equations (**PDEs**)

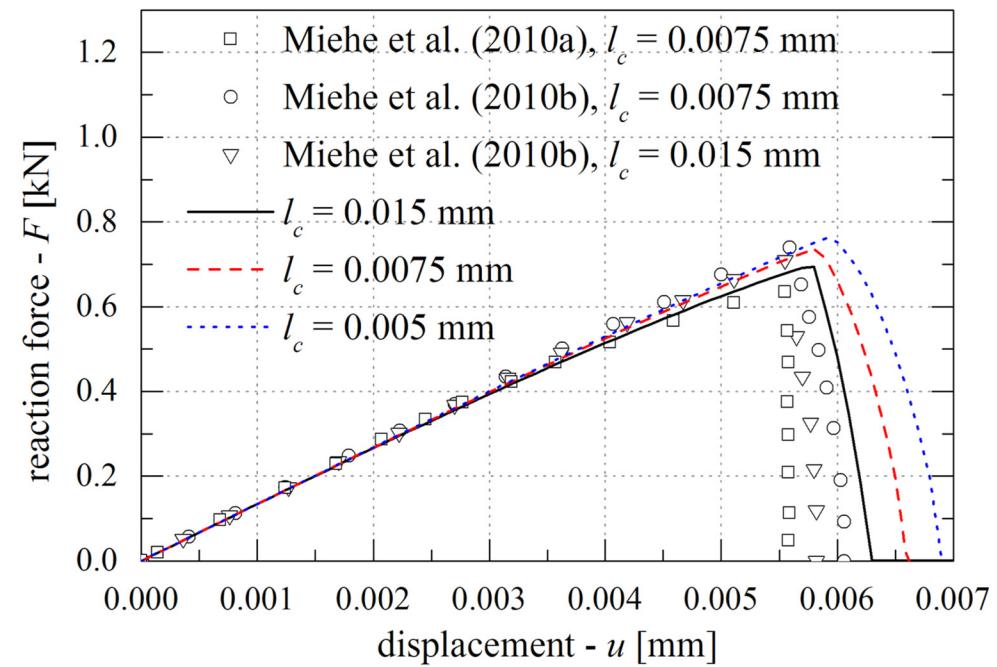
Parameters

Single notched specimen under tension

(a) geometry [mm]

(b) crack pattern ($\alpha = 90^\circ$)
pure tension

$$\Delta l_c = 300\% \rightarrow \Delta F_{\max} \approx 10\%$$

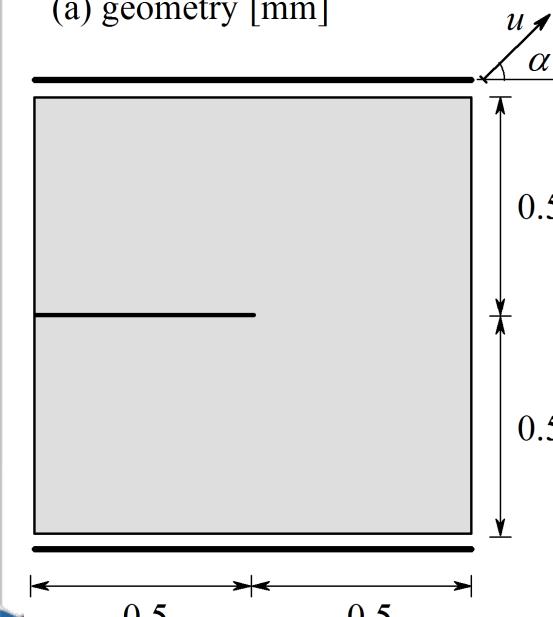
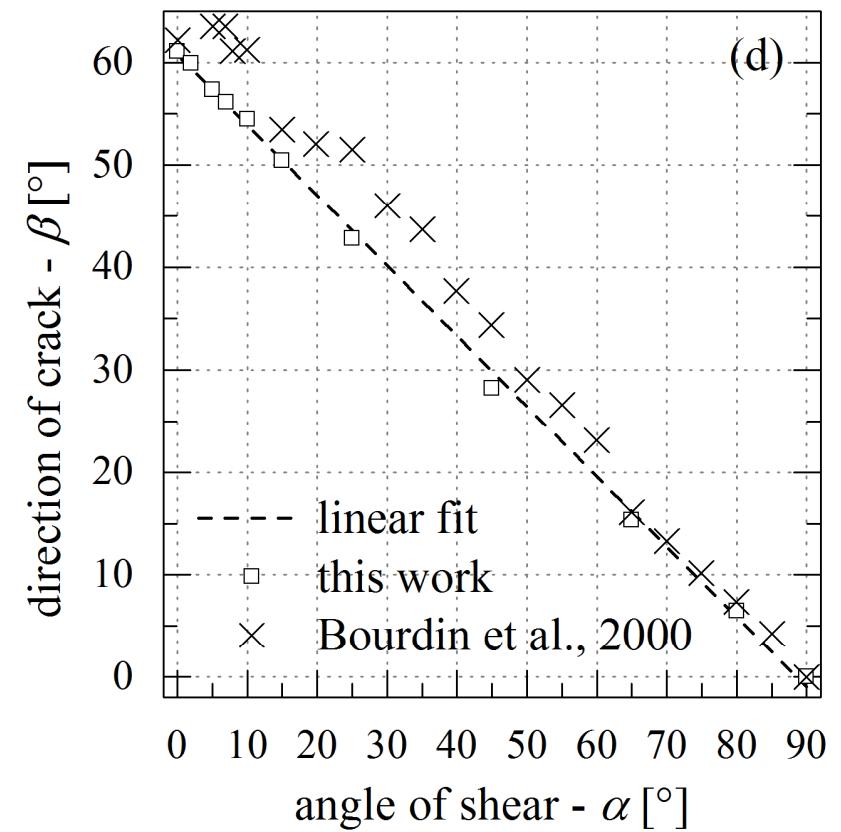
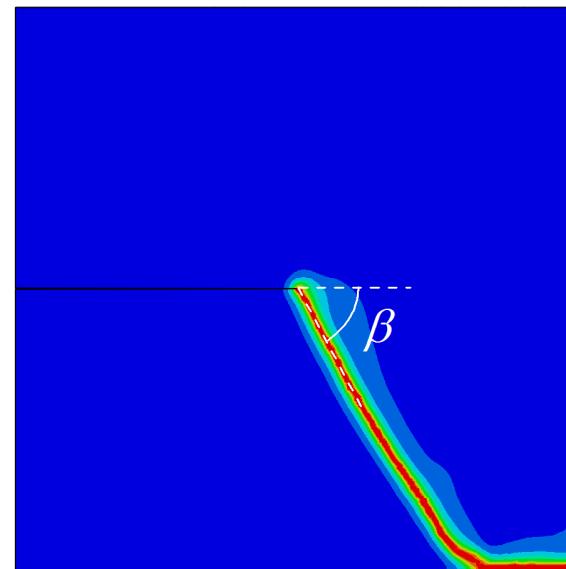


Effect of **length-scale**

Parameters

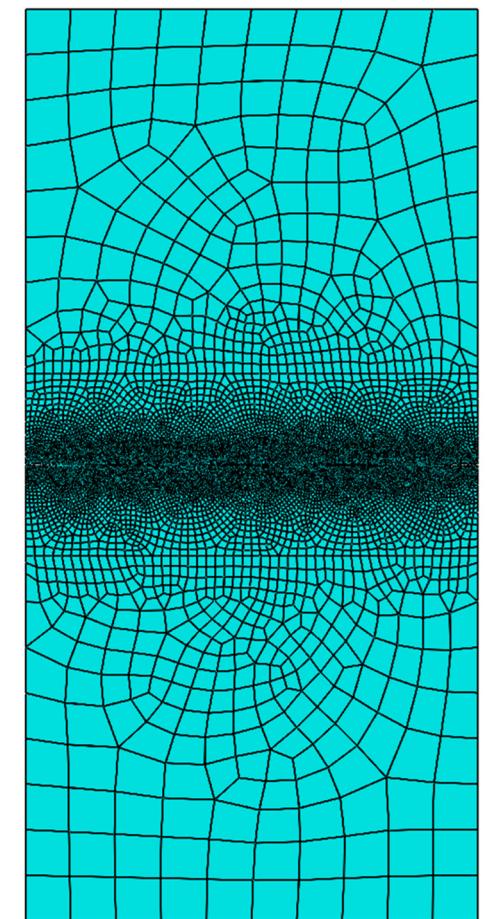
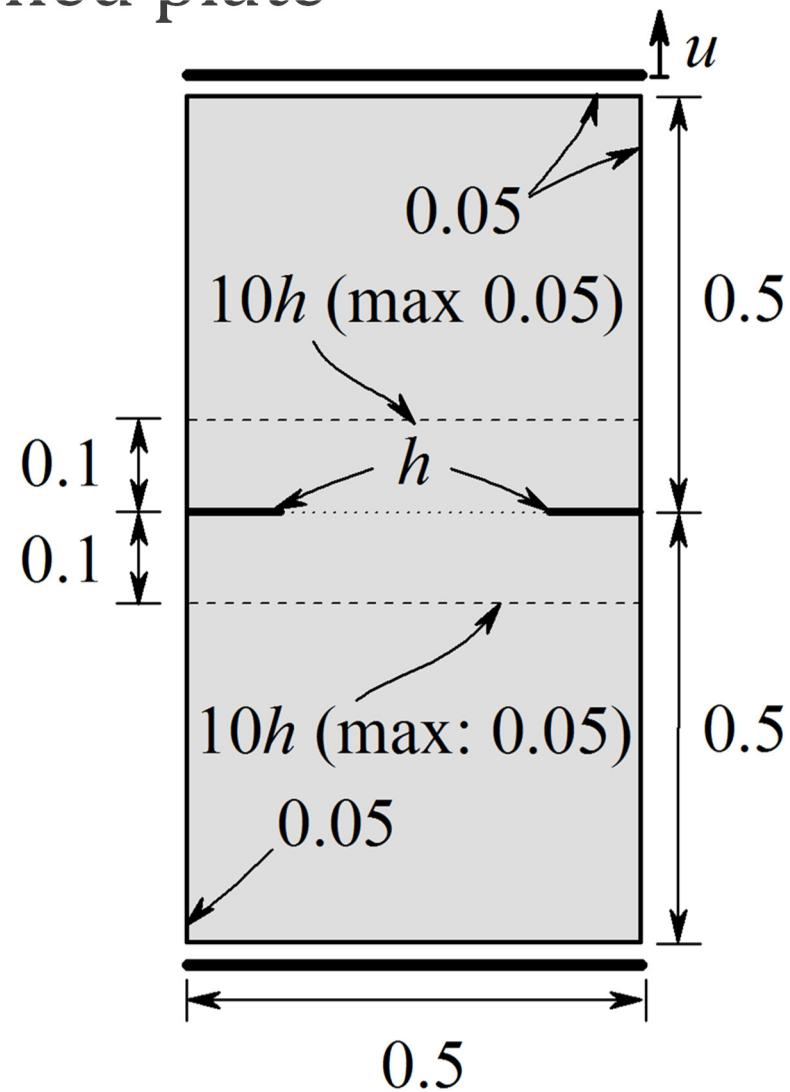
Single notched specimen under shear

(a) geometry [mm]

(c) crack pattern ($\alpha = 0^\circ$)
pure shear

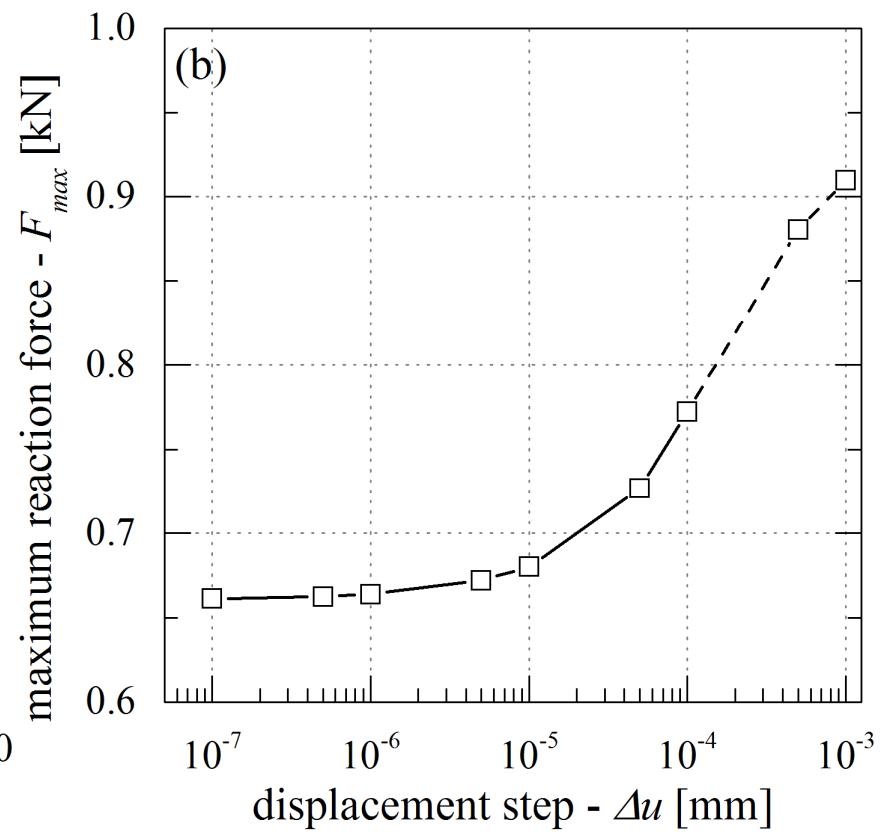
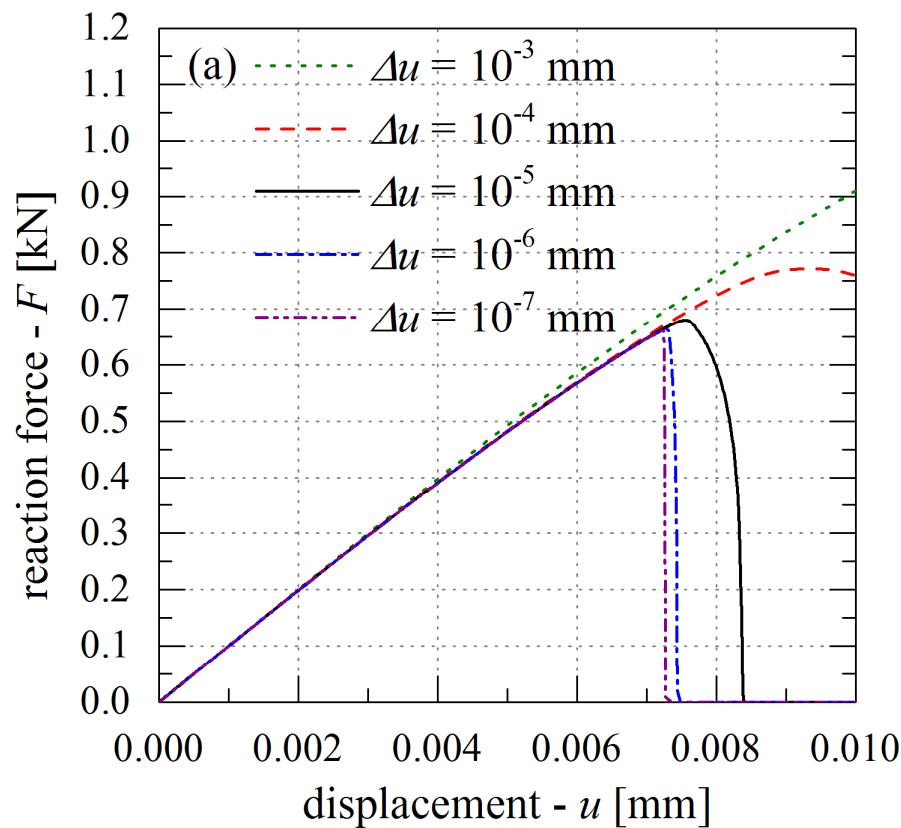
Parameters

Double notched plate



Parameters

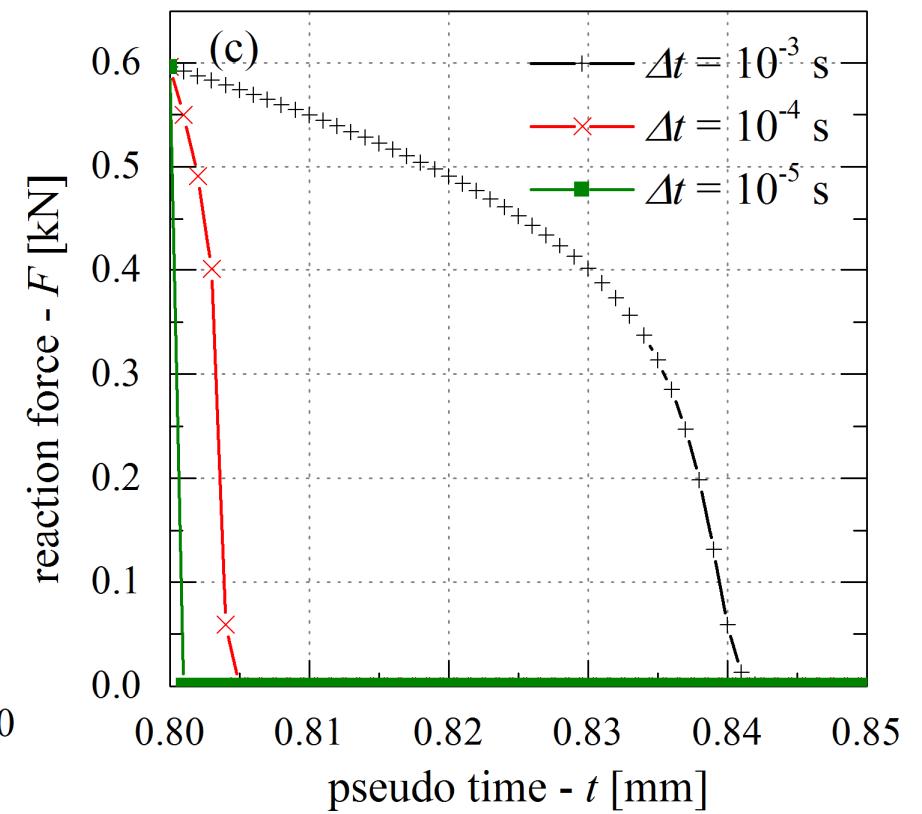
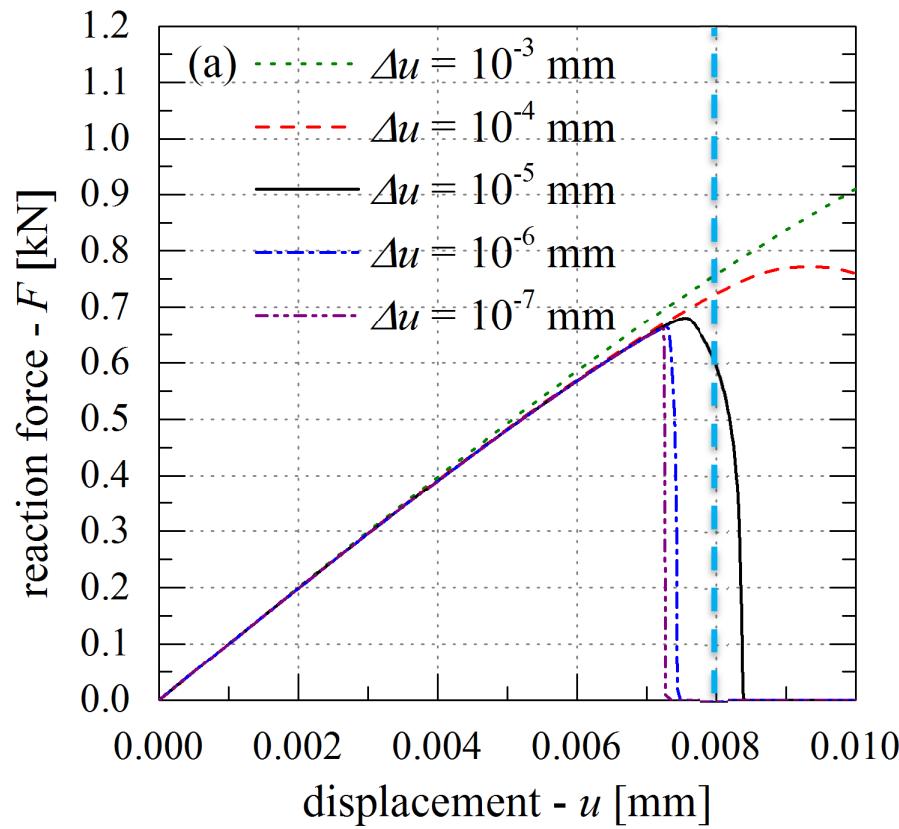
Time step



Parameters

Time step

Deformation is applied until 0.008 mm then stopped

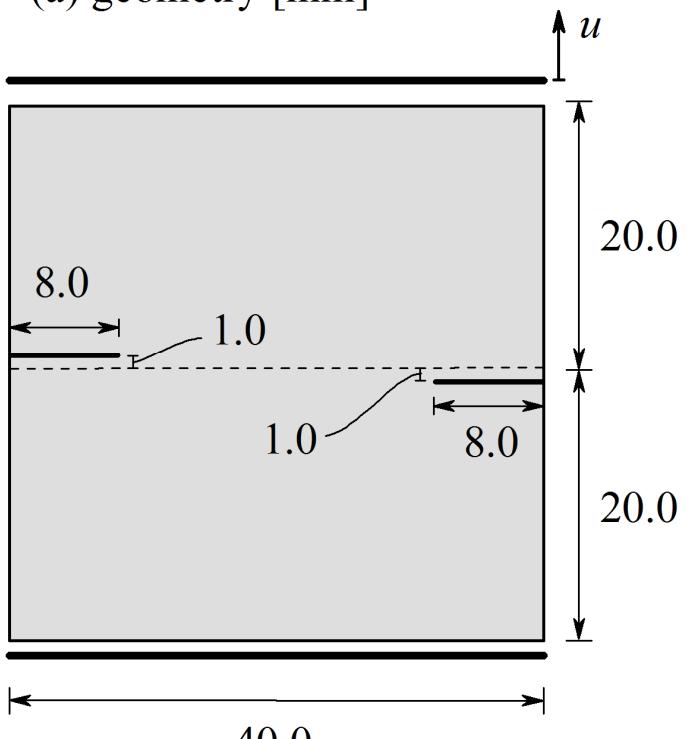


For details see **Tutorial 3: Cracked cylinder in tension** on
www.molnar-research.com

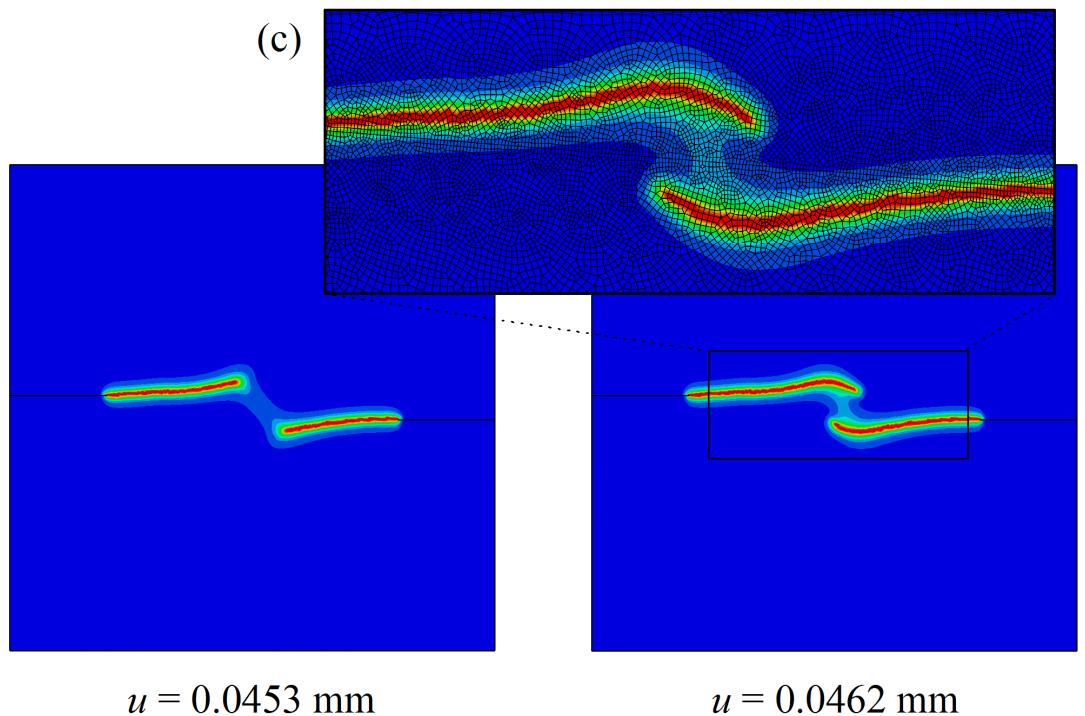
Examples

Asymmetric double notched plate

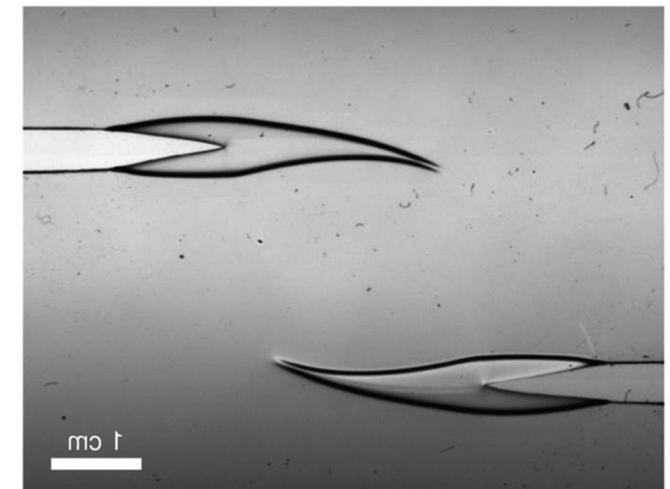
(a) geometry [mm]



Réthoré et al., 2010



$u = 0.0462 \text{ mm}$

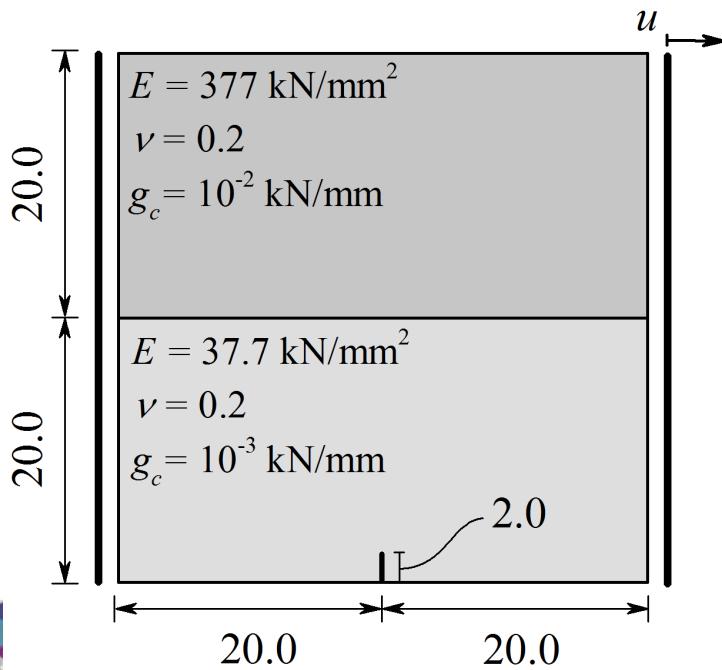


Koivisto et al., 2016

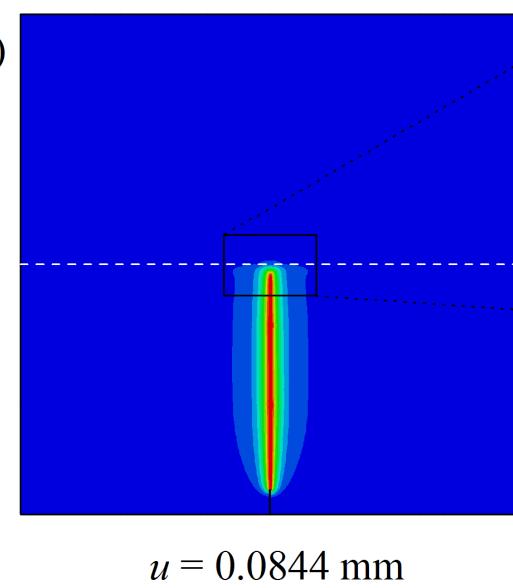
Examples

Bi-material tension

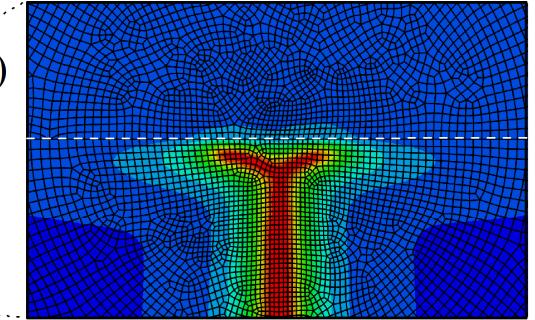
(a) geometry



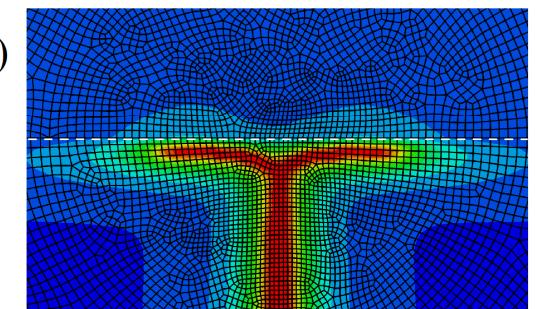
(b)



(c)



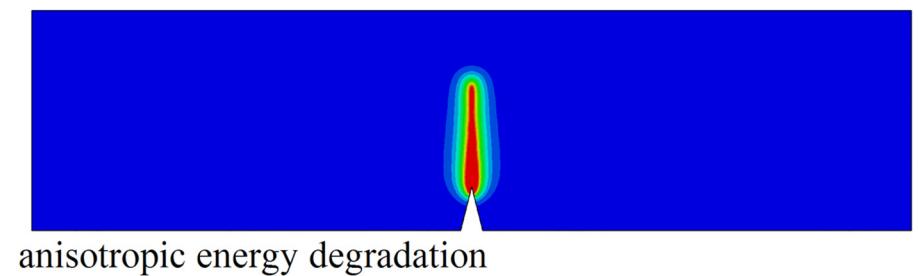
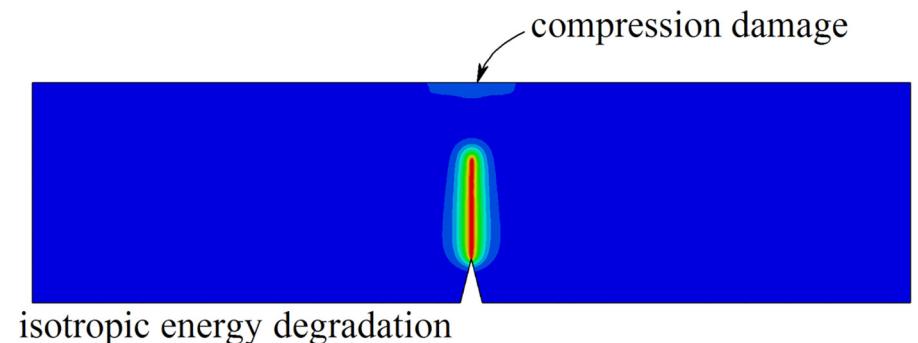
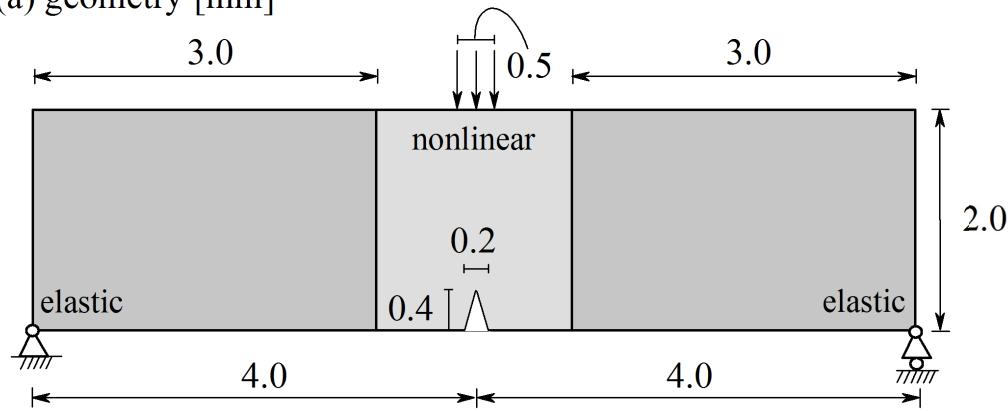
(d)

 $u = 0.1144 \text{ mm}$ $u = 0.1300 \text{ mm}$

Examples

Positive and negative energy degradation

(a) geometry [mm]



$$\begin{aligned} \psi_0^\pm = & \frac{E\nu}{(1+\nu)(1-2\nu)} \left\langle \text{tr}(\boldsymbol{\varepsilon}) \right\rangle_\pm^2 + \\ & + \frac{E}{2(1+\nu)} \left(\left\langle \boldsymbol{\varepsilon}_2 \right\rangle_\pm^2 + \left\langle \boldsymbol{\varepsilon}_2 \right\rangle_\pm^2 \right) \end{aligned}$$

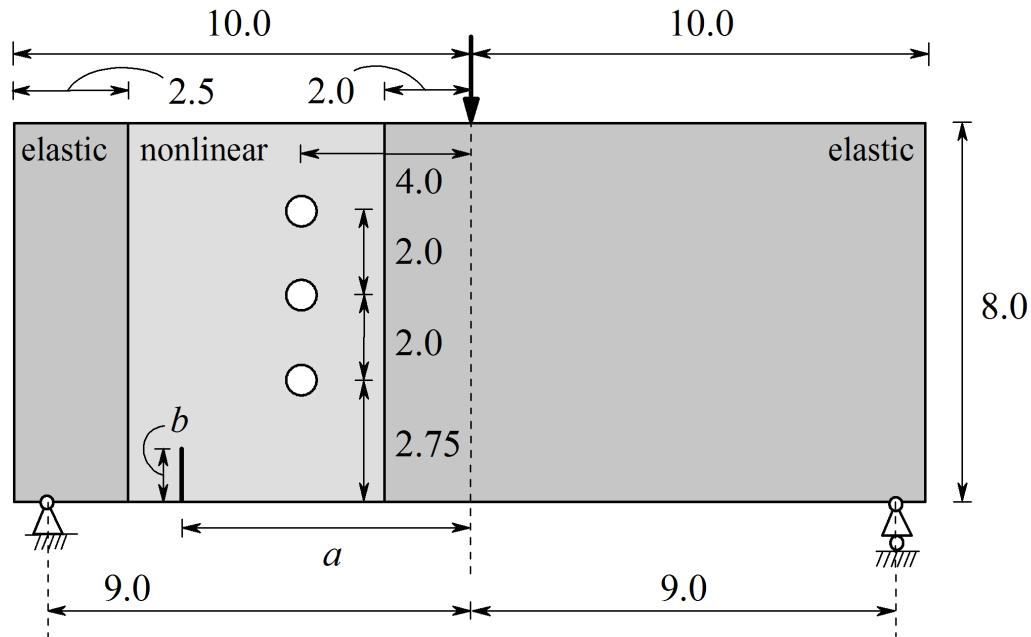
$$\psi = \psi_0^+ g(d) + \psi_0^-$$

Examples

Asymmetric bending

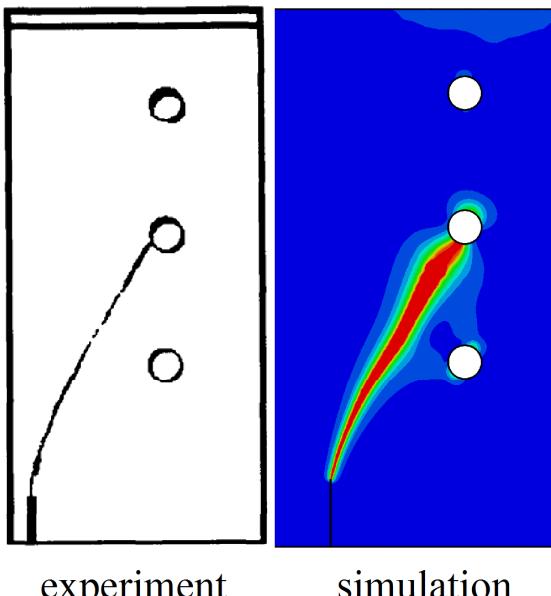
Bittencourt et al., 1996

(a) geometry [mm]



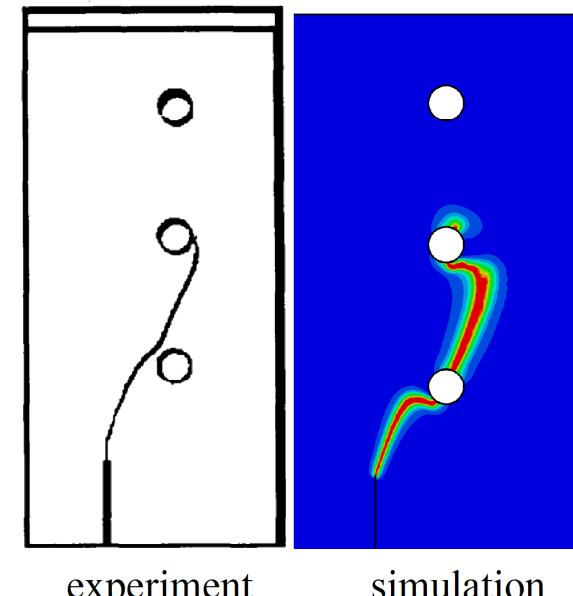
Example 1

($a = 6.0$ mm, $b = 1.0$ mm)



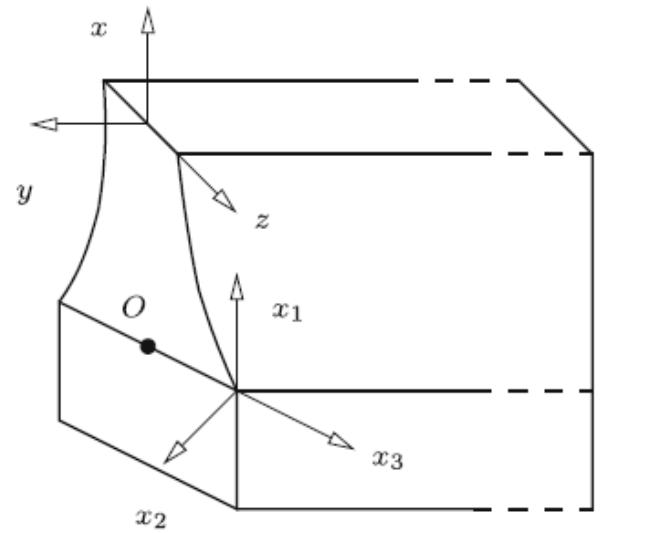
Example 2

($a = 4.0$ mm, $b = 1.5$ mm)

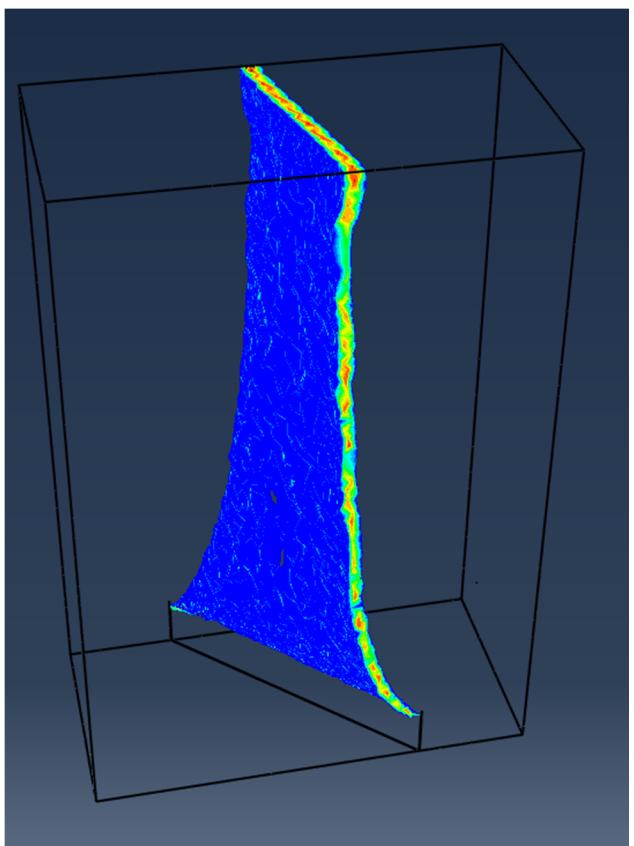
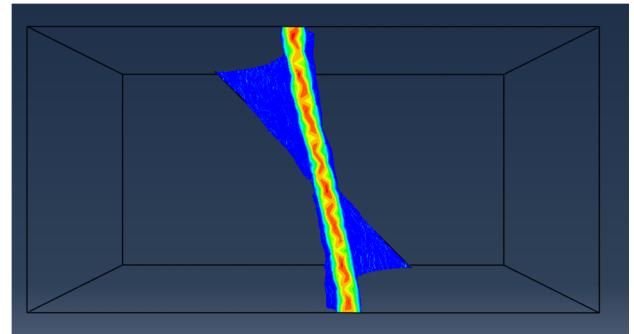
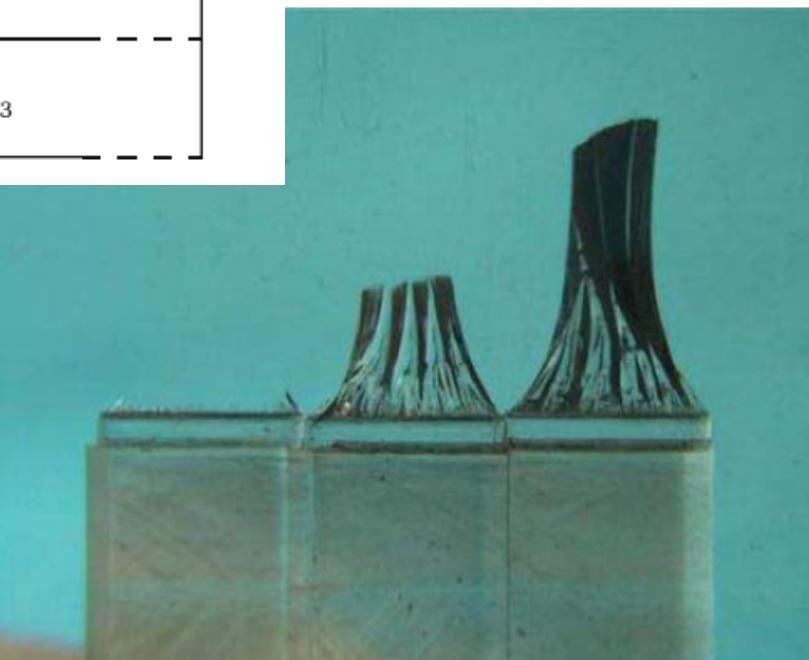


3D Examples

Inclined crack in bending

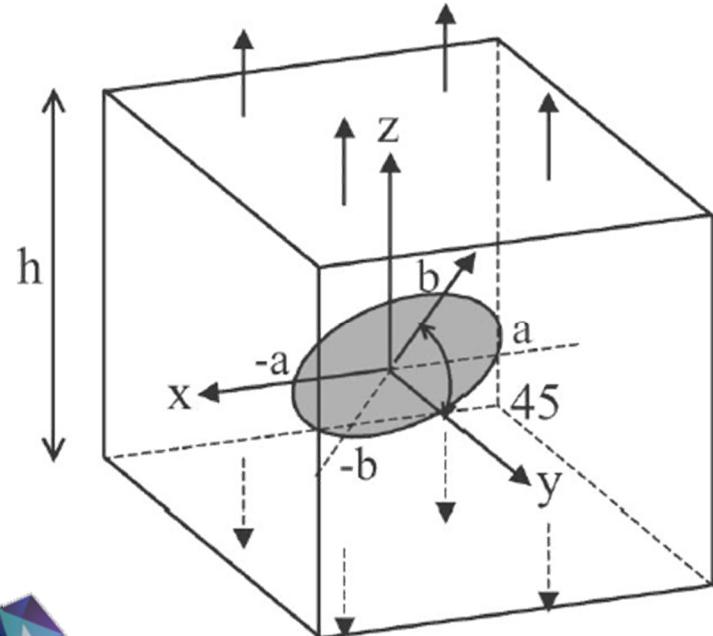


Lazarus et al., 2008

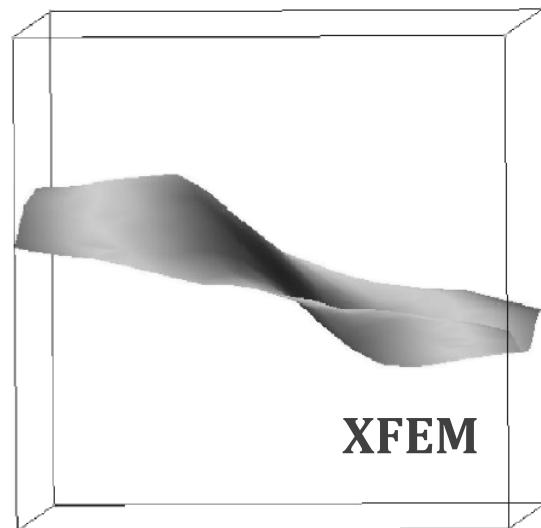
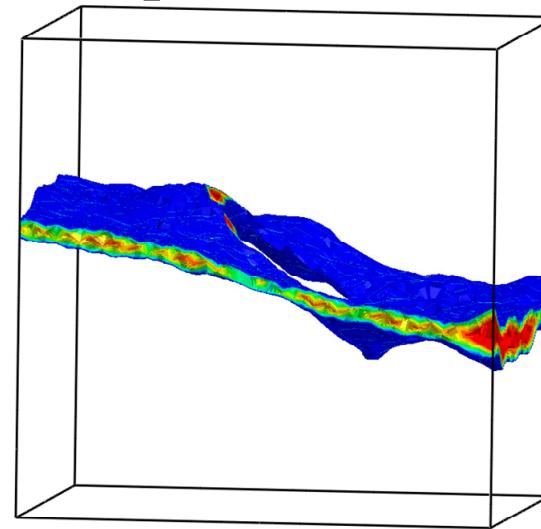


3D Examples

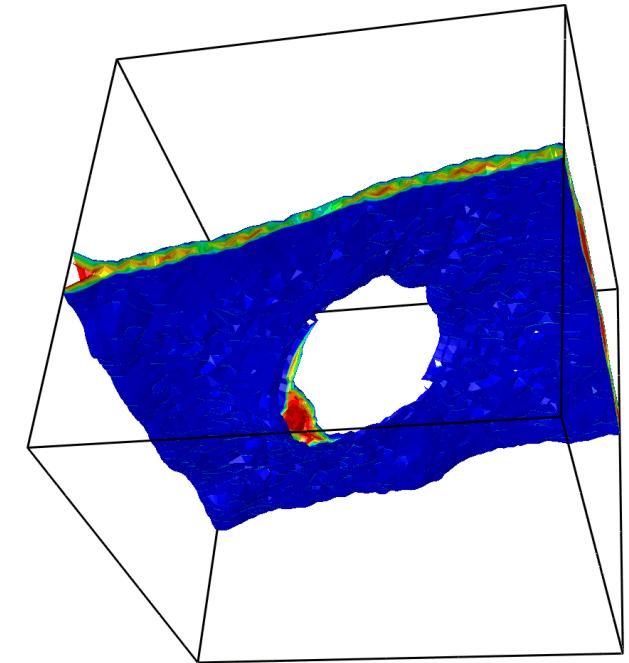
Inclined penny shape crack in tension



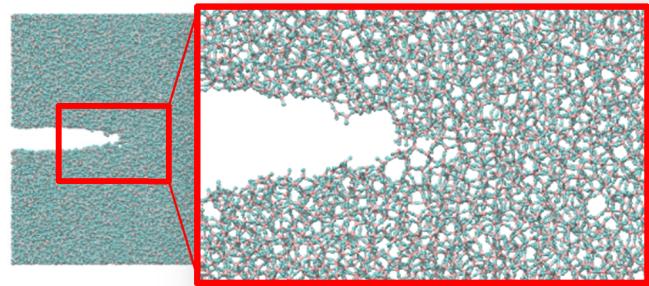
Gravouil et al., 2002



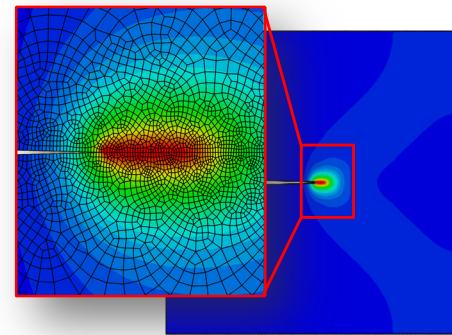
XFEM



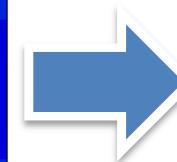
Conclusion



Microstructure



Phase-field



Hybrid FDEM
XFEM/GFEM
Cohesive Zones

Predefined crack

Advantages and disadvantages

- crack initiation, propagation
- branching, merging
- fixed mesh
- fully 3D



- fine mesh
- finite crack size
- efficiency/robustness

Versatility

dynamics, shells, nonlinear elasticity, large strains,
coupled problems, plasticity, anisotropy, etc...

Where to find it?

Examples and tutorials: www.molnar-research.com

The screenshot shows a web browser window for 'Gergely Molnár' at 'www.molnar-research.com'. The navigation bar includes links for HOME, ABOUT ME, RESEARCH (with a dropdown arrow), TUTORIALS (which is highlighted with a red box), PUBLICATIONS, CONTACT, and LINKS. Below the navigation bar, there are three 3D models: a white cube with dimensions 5.0x5.0x5.0, a teal cube with a complex internal mesh, and a white cube with a red diffuse surface. A large red 'G' is visible on the right side of the page. A sidebar on the right contains the following text and diagram:

1. Simple tension with 2 elements
The tutorial presents a simple conversion between the input file generated by ABAQUS and the use of the new UEL.
The instructions can be downloaded from [here](#). While the files used and created through the tutorial are accessible from [this link](#).

The diagram illustrates a mechanical system for a simple tension test. It consists of a central vertical element supported by two horizontal bars. The top horizontal bar has a displacement of $du = 0.1$ indicated by an arrow. The bottom horizontal bar is fixed to a base. The total height of the central element is labeled as $2 \times 0.50 = 1.00$. A coordinate system with axes x and y is shown.

**FORTRAN files
INPUT files
Tutorials**

References

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- J.-D. Wörner, Glasbau, 2001.

Thank you for your attention

Now let's try it out!

Please visit my homepage

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gmolnar.work@gmail.com

