Numerical modeling of brittle fracture using the phase-field method

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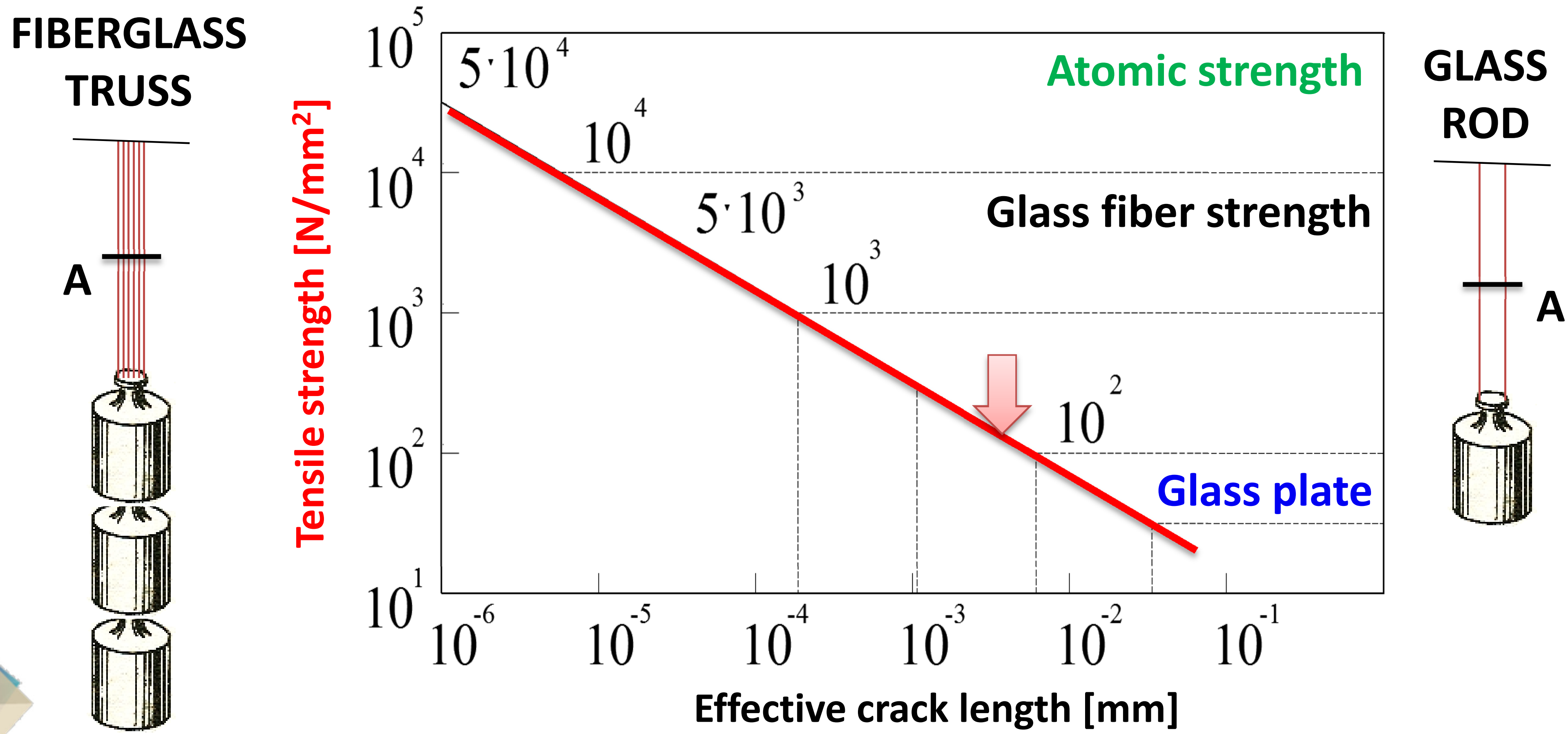
3SR

Introduction

Macroscopic strength

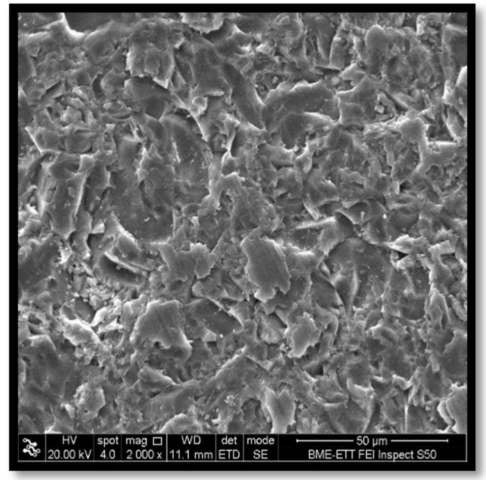


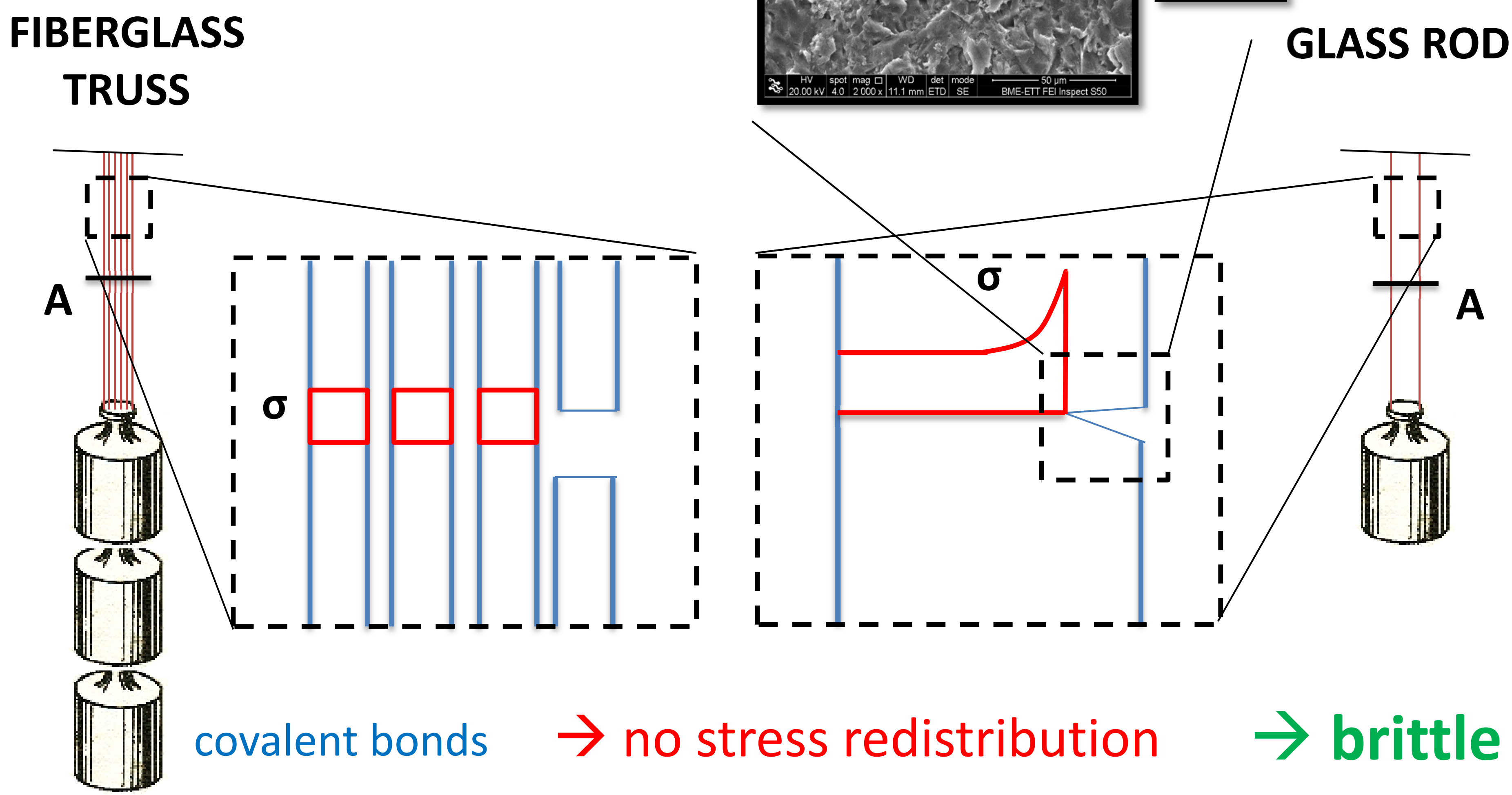
Wörner, 2001.

Molnár et al., 2012, 2013.

Introduction

Macroscopic strength



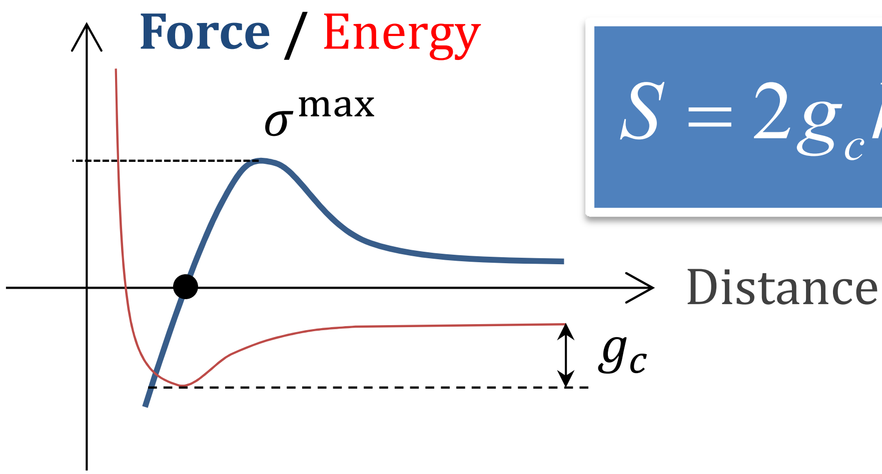


covalent bonds → no stress redistribution → **brittle**

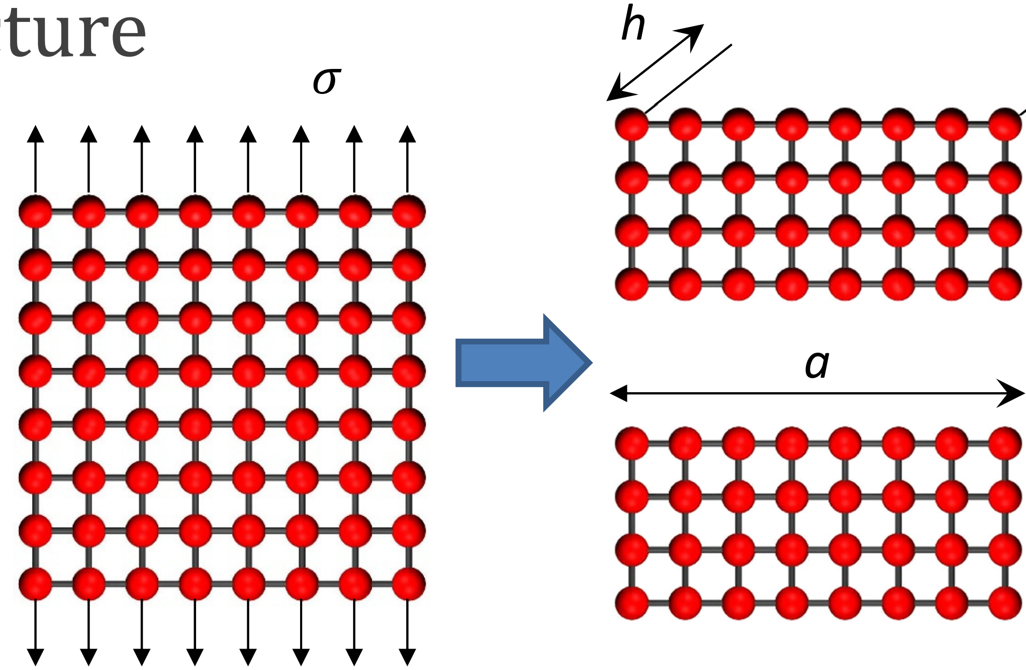
Griffith, 1920

Introduction

Griffith theory of brittle fracture



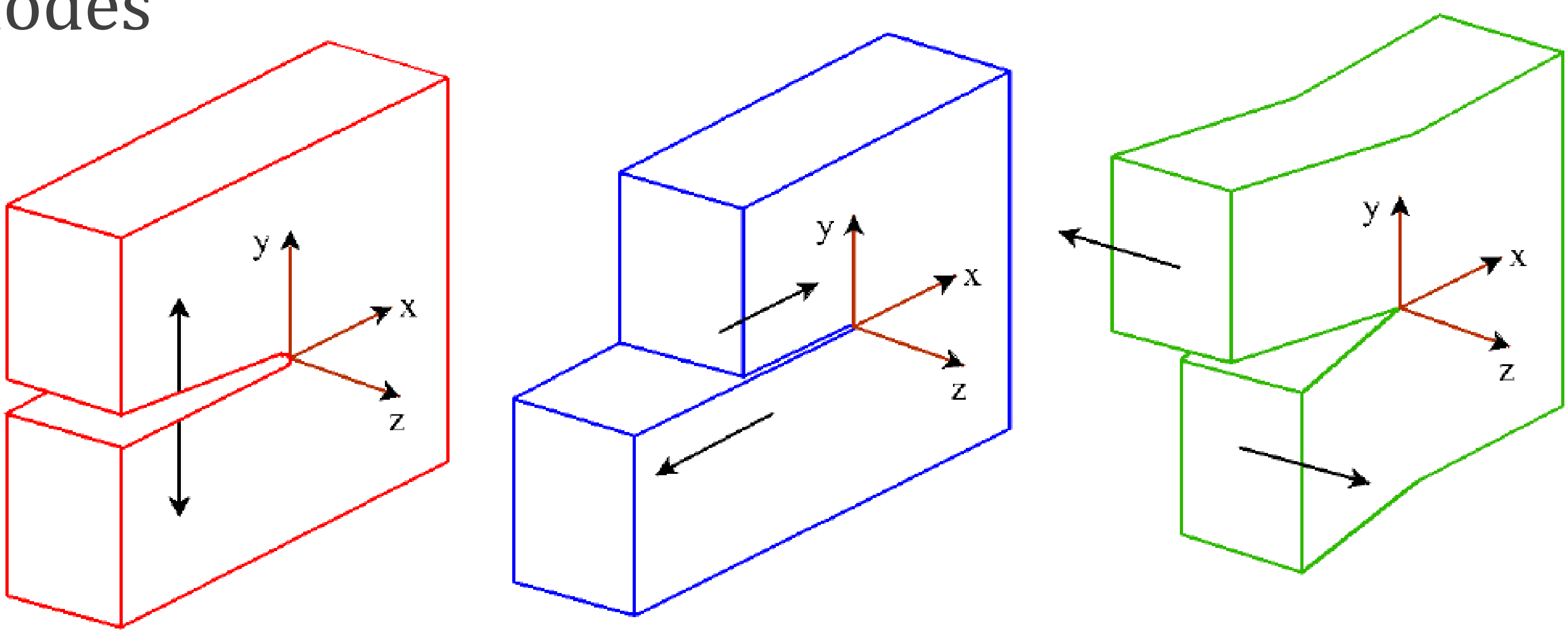
$$S = 2g_c ha$$



Crack modes

Irwin, 1957

Erdogan , 2000



Introduction

How do we approximate it with a phase-field?

1. Brittle fracture
$$-\frac{\partial \psi}{\partial a} = \frac{\partial S}{\partial a} = g_c$$

Griffith, 1920

2. Minimization problem
$$E(\mathbf{u}, \Gamma) = \int_\Omega \psi(\varepsilon(\mathbf{u}))\, d\Omega + g_c \int_\Gamma d\Gamma$$

Mumford & Shah, 1989

Francfort & Marigo, 1998

3. Crack energy density
$$E(\mathbf{u}, d) = \int_\Omega g(d)\psi(\varepsilon(\mathbf{u}))\, d\Omega + g_c \int_\Omega \left(\frac{1}{2l_c} d^2 + \frac{l_c}{2} |\nabla d|^2 \right) d\Omega$$

$l_c \to 0$ Γ converges

$\dot{d} \geq 0$

crack energy density - γ

Ambrosio & Tortorelli, 1990

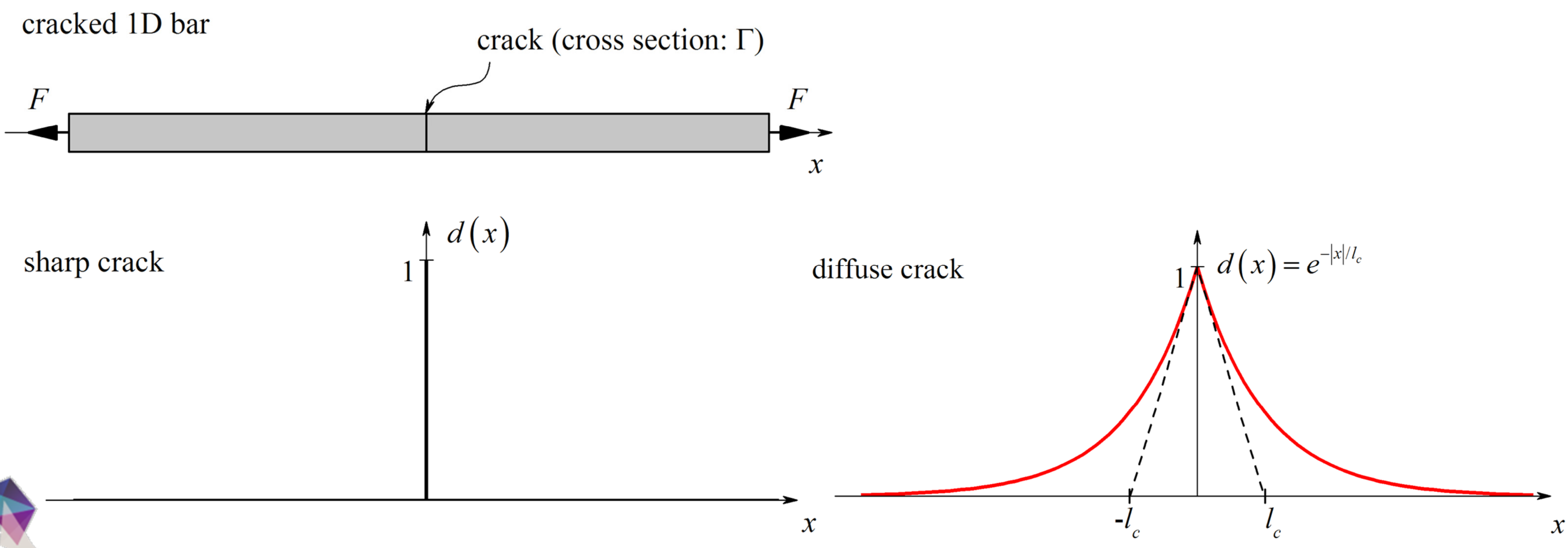
Bourdin et al., 2000

Amor et al., 2009

Miehe et al., 2010a

Introduction

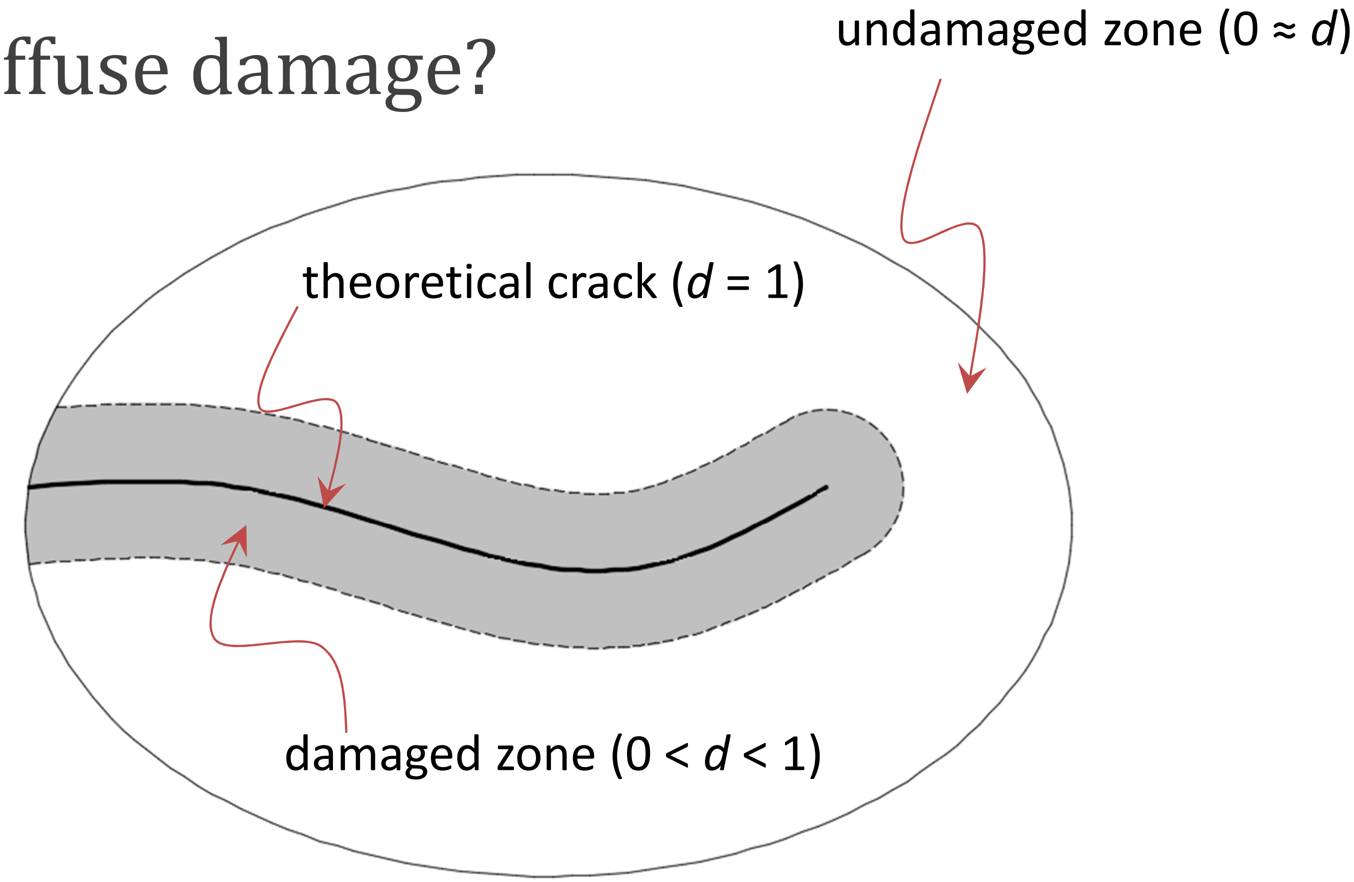
What is diffuse damage?



Solving **fracture mechanics** problem with Partial Differential Equations (**PDE**s)

Introduction

What is diffuse damage?



Solving **fracture mechanics** problem with Partial Differential Equations (**PDE**s)

Phase-field method

Prof. Dr.-Ing.
Christian Miehe
(1956 – †2016)

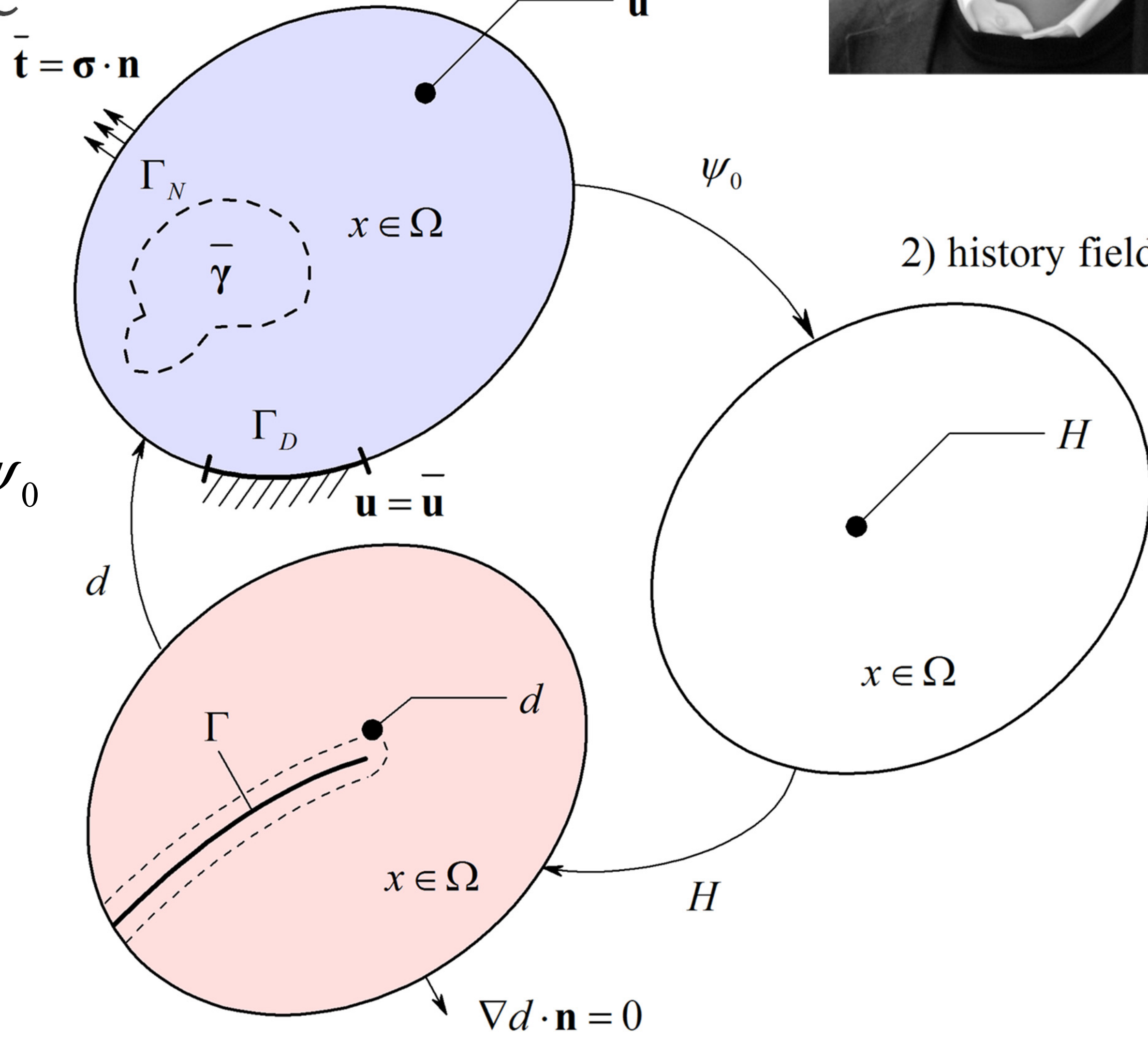
Staggered scheme

Miehe et al., 2010b

$$E^u(\mathbf{u}, d)$$

$$E^d(d, H_n)$$

$$H_n = \psi_0(\varepsilon(\mathbf{u})) \quad \text{if} \quad H_{n-1} < \psi_0$$



Robustness!!!

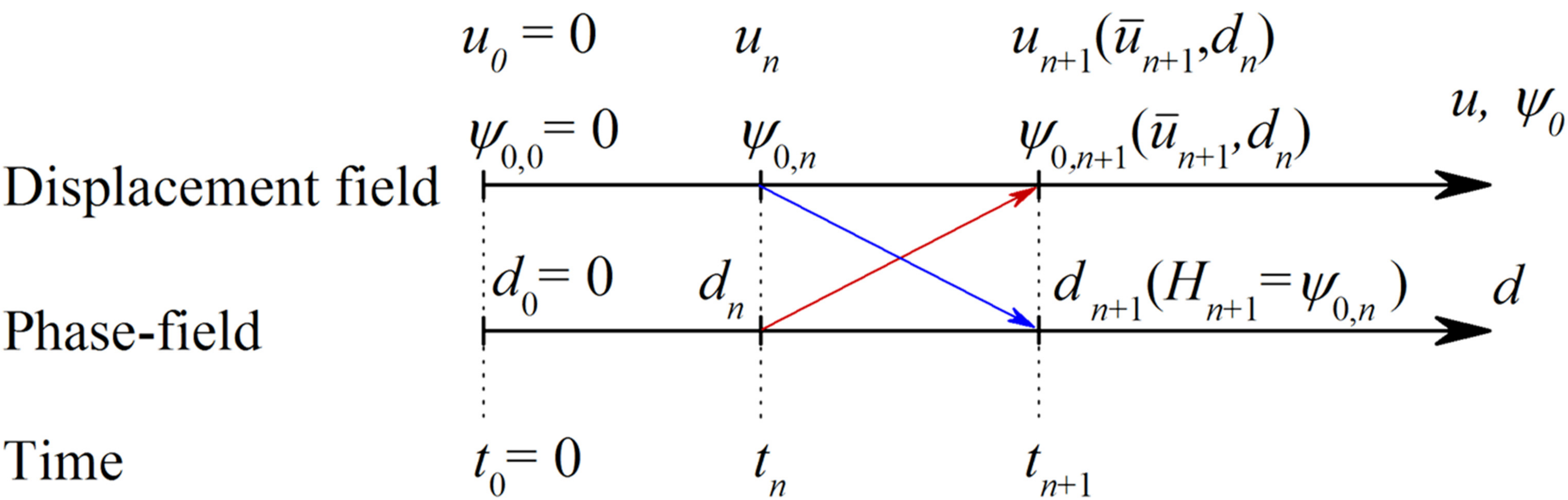
Efficiency?

Molnár & Gravouil, 2017

Phase-field method

Staggered scheme

Prof. Dr.-Ing.
Christian Miehe
(1956 – †2016)



$$E^u(\mathbf{u}, d)$$

$$E^d(d, H_n)$$

$$H_n = \psi_0(\varepsilon(\mathbf{u})) \quad \text{if} \quad H_{n-1} < \psi_0$$

Molnár & Gravouil, 2017

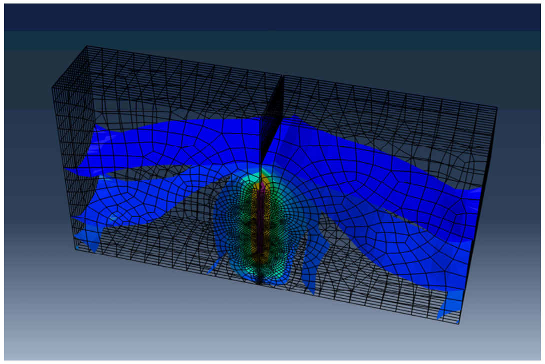
Open source implementation

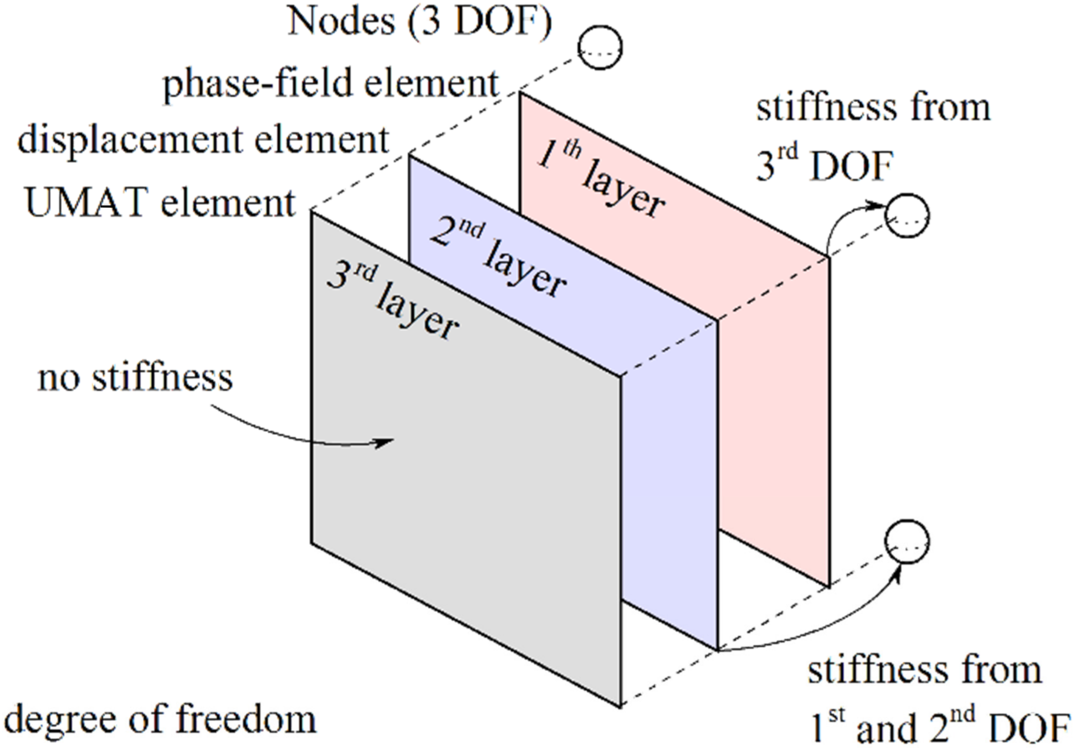
ABAQUS/UEL option (ABAQUS + FORTRAN compiler)

stiffness matrix + **residue vector** for every element

Visualization

FORTRAN and ABAQUS files are available in both **2D** and **3D**





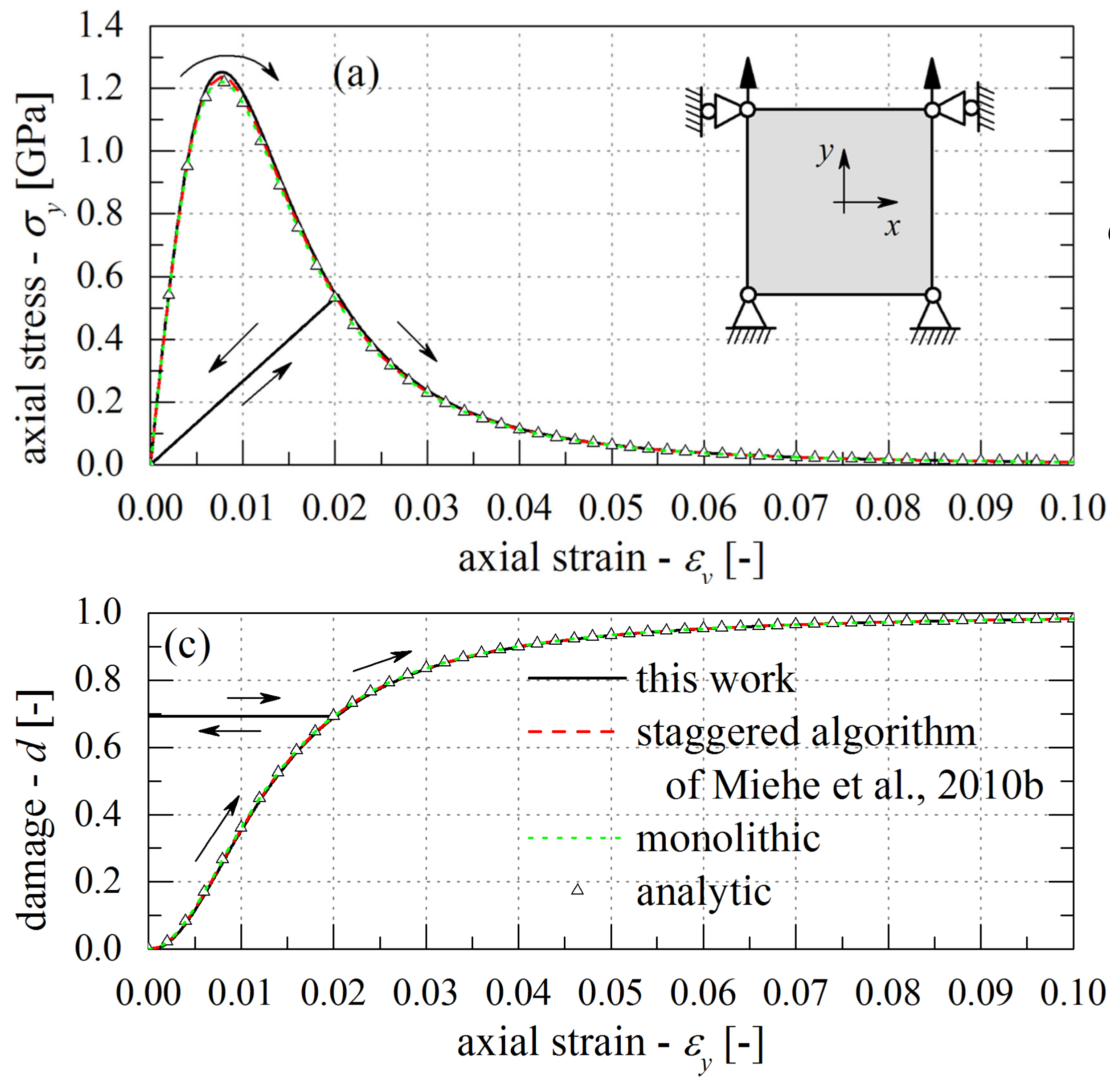
DOF - degree of freedom

Molnár & Gravouil, 2017

Phase-field method

c_{22} - (2,2) element of the stiffness matrix

Single element solution



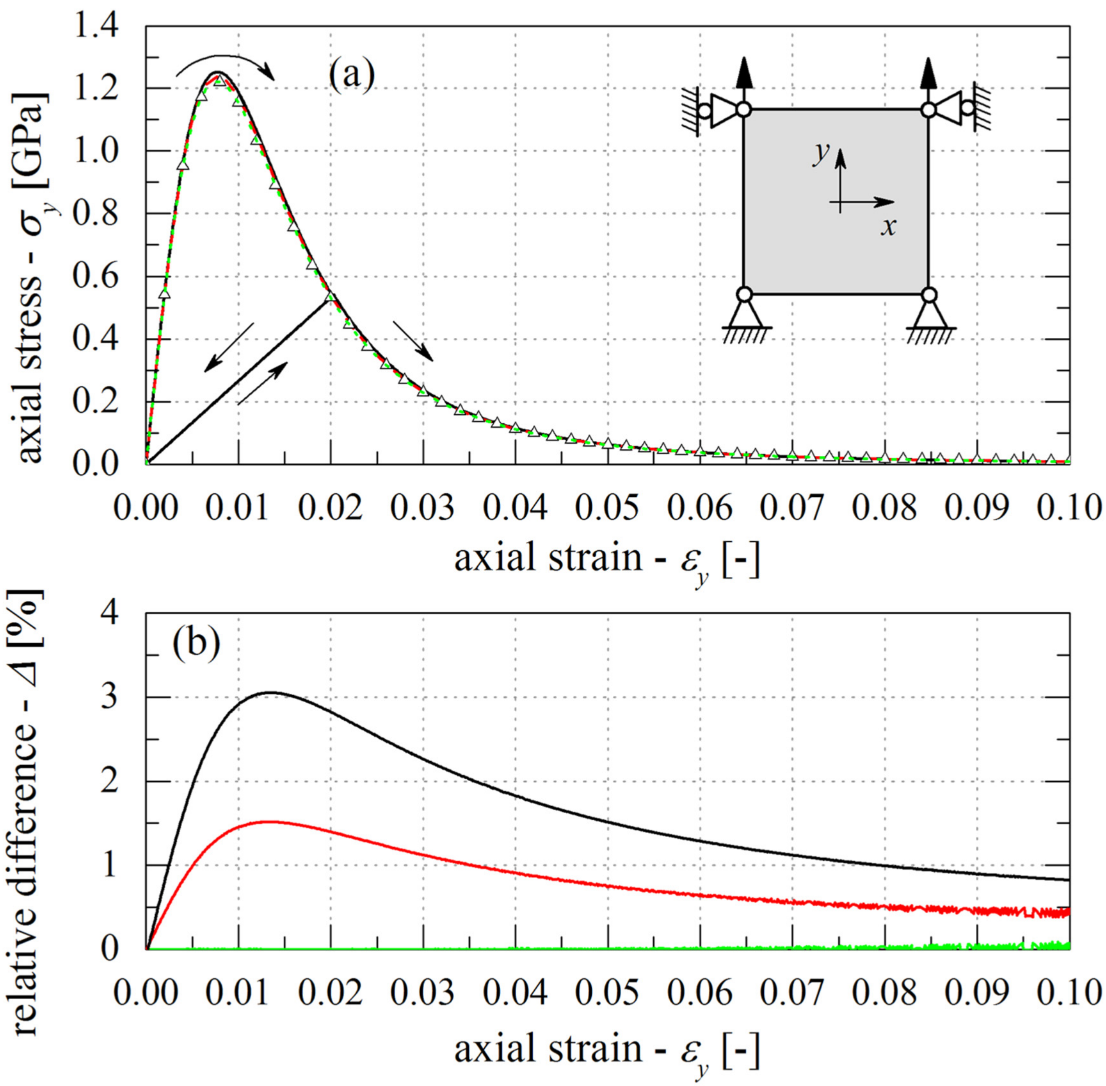
$$\sigma_y = (1-d)^2 \varepsilon_y c_{22}$$

$$d = \frac{\varepsilon_y^2 c_{22}}{\frac{g_c}{l_c} + \varepsilon_y^2 c_{22}}$$

Molnár & Gravouil, 2017

Phase-field method

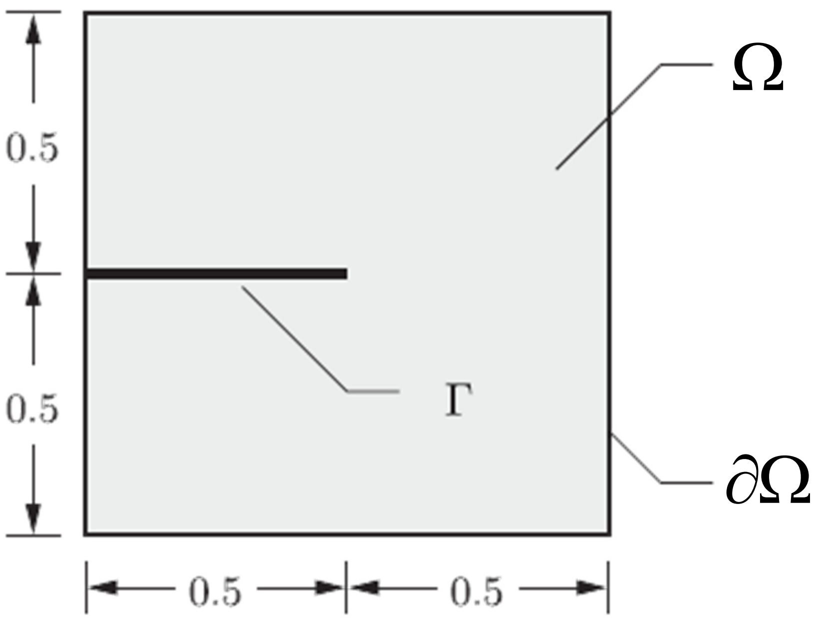
Single element solution

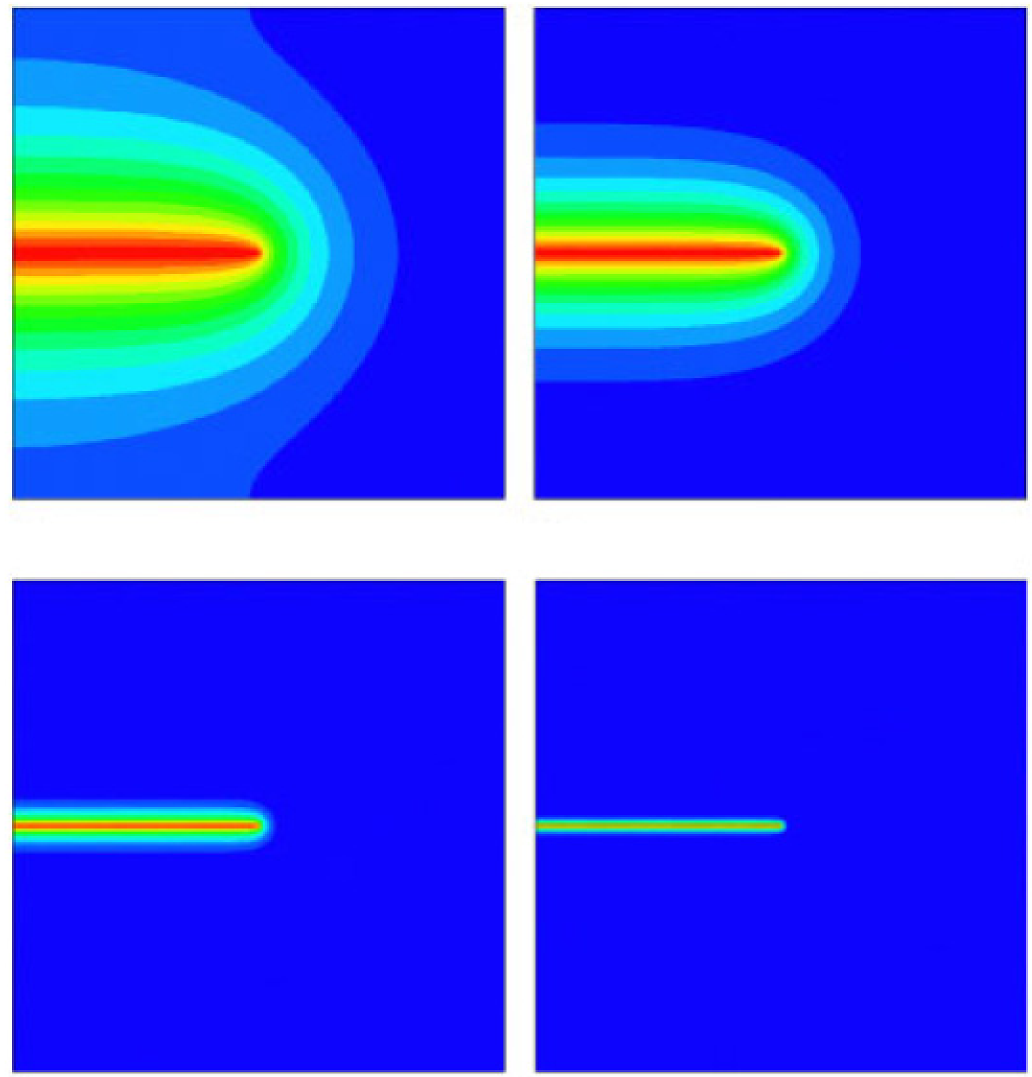


$$\Delta = \frac{\sigma_y - \sigma_y^{\text{analytic}}}{\sigma_y^{\text{analytic}}}$$

Parameters

How fine should the mesh be?





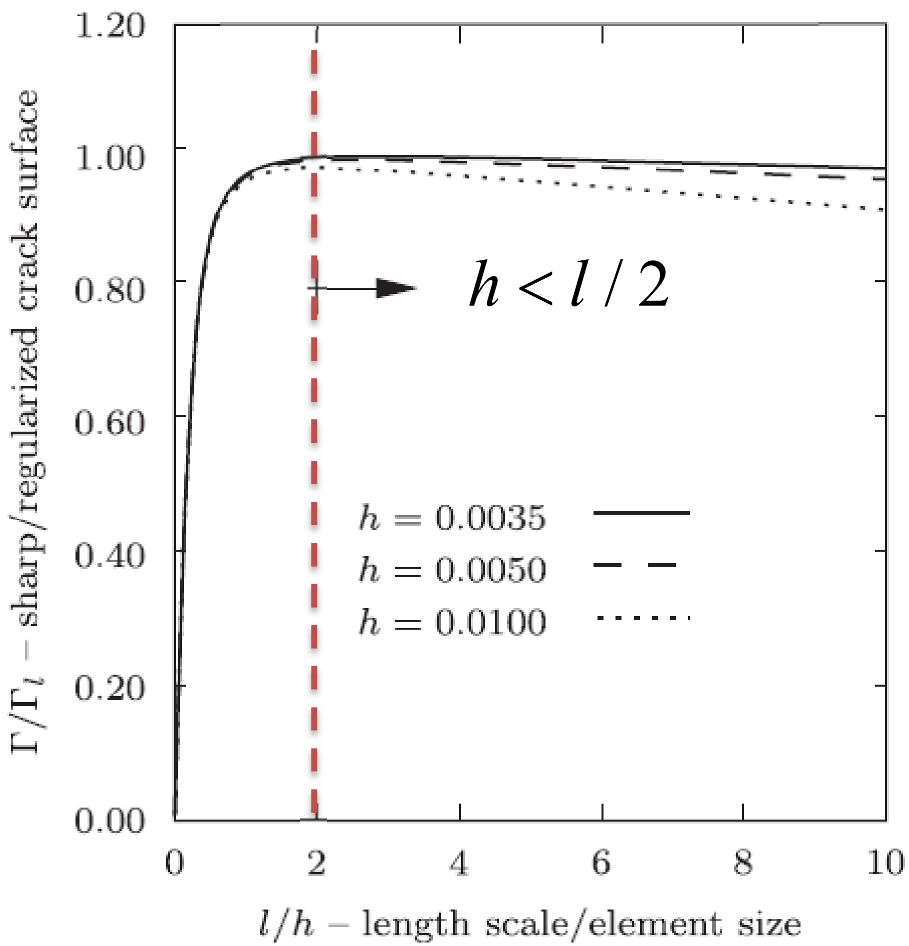
Miehe et al., 2010a

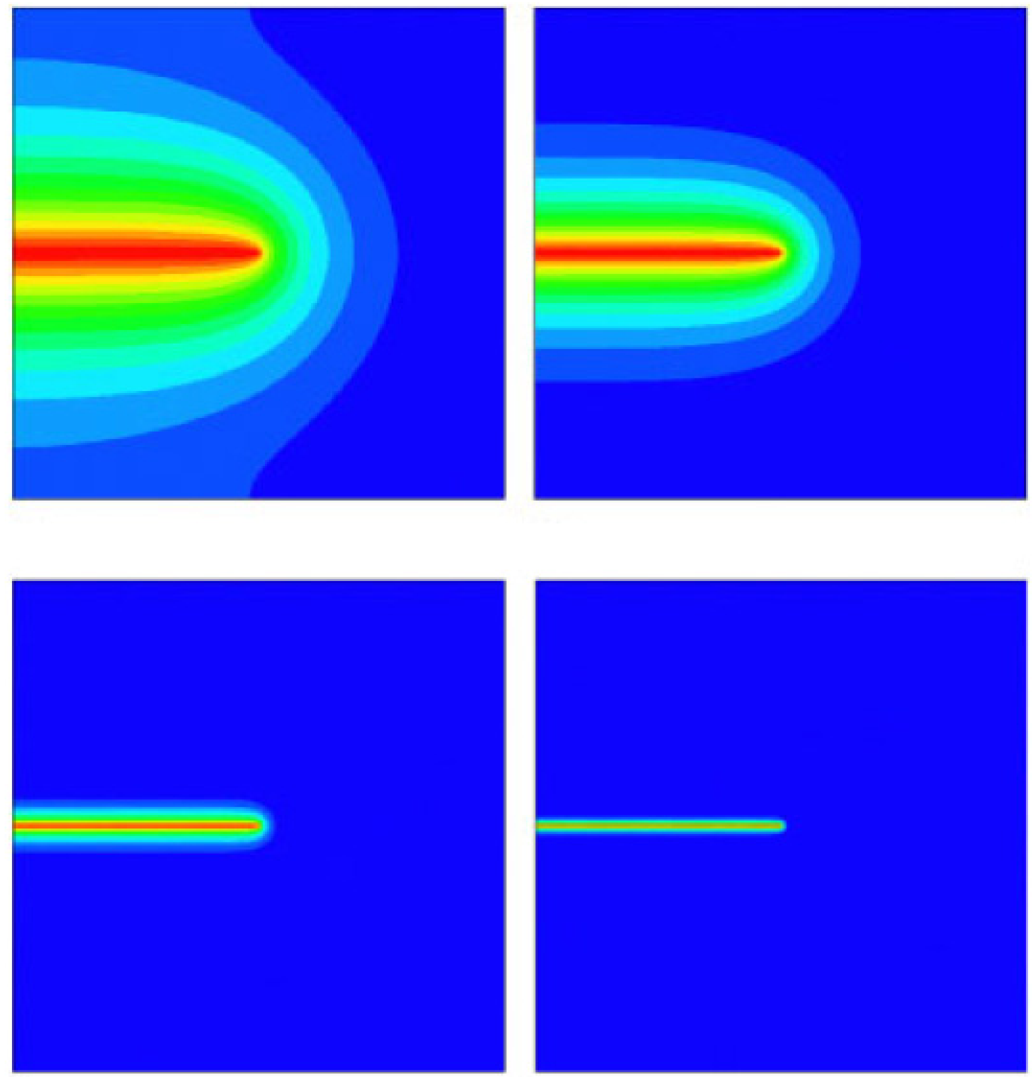
Parameters

theoretical → $\Gamma = 0.5$

$$\Gamma_l = \int_\Omega \gamma \, d\Omega$$

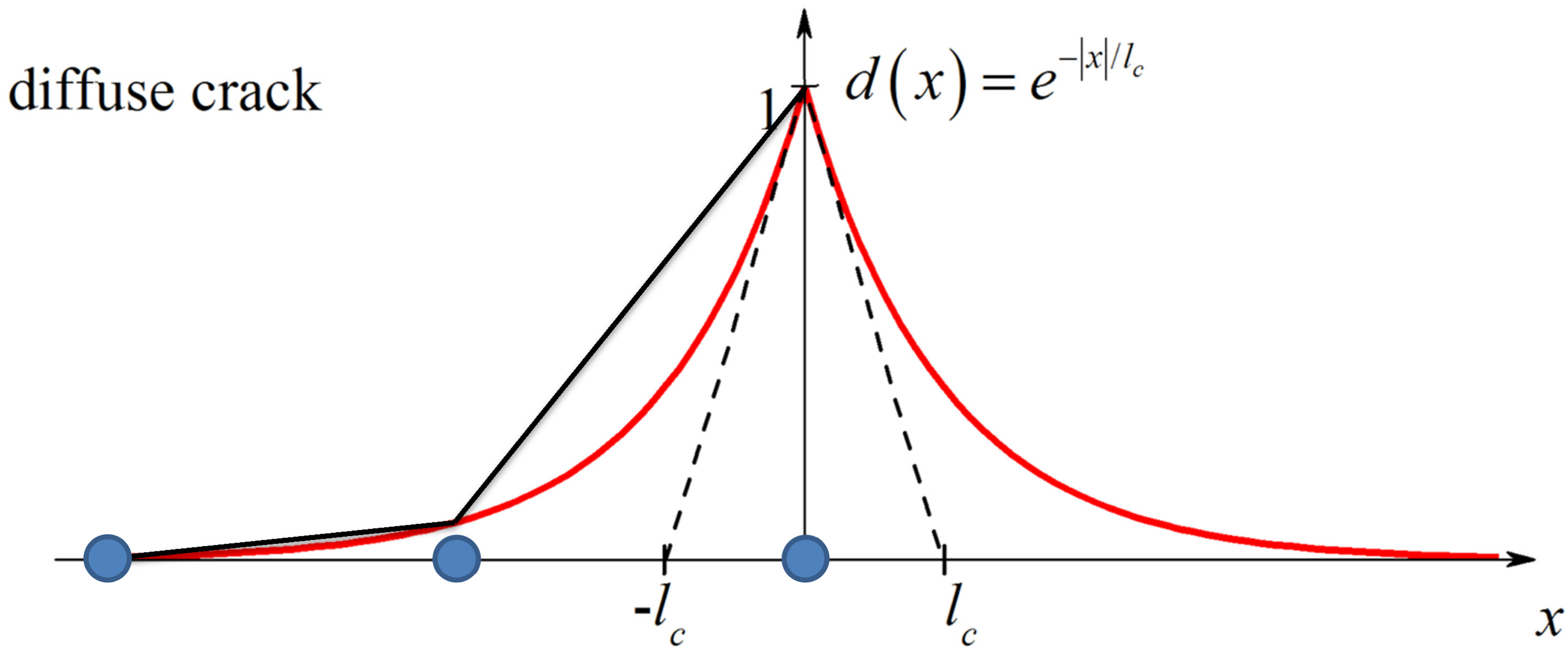
How fine should the mesh be?





Parameters

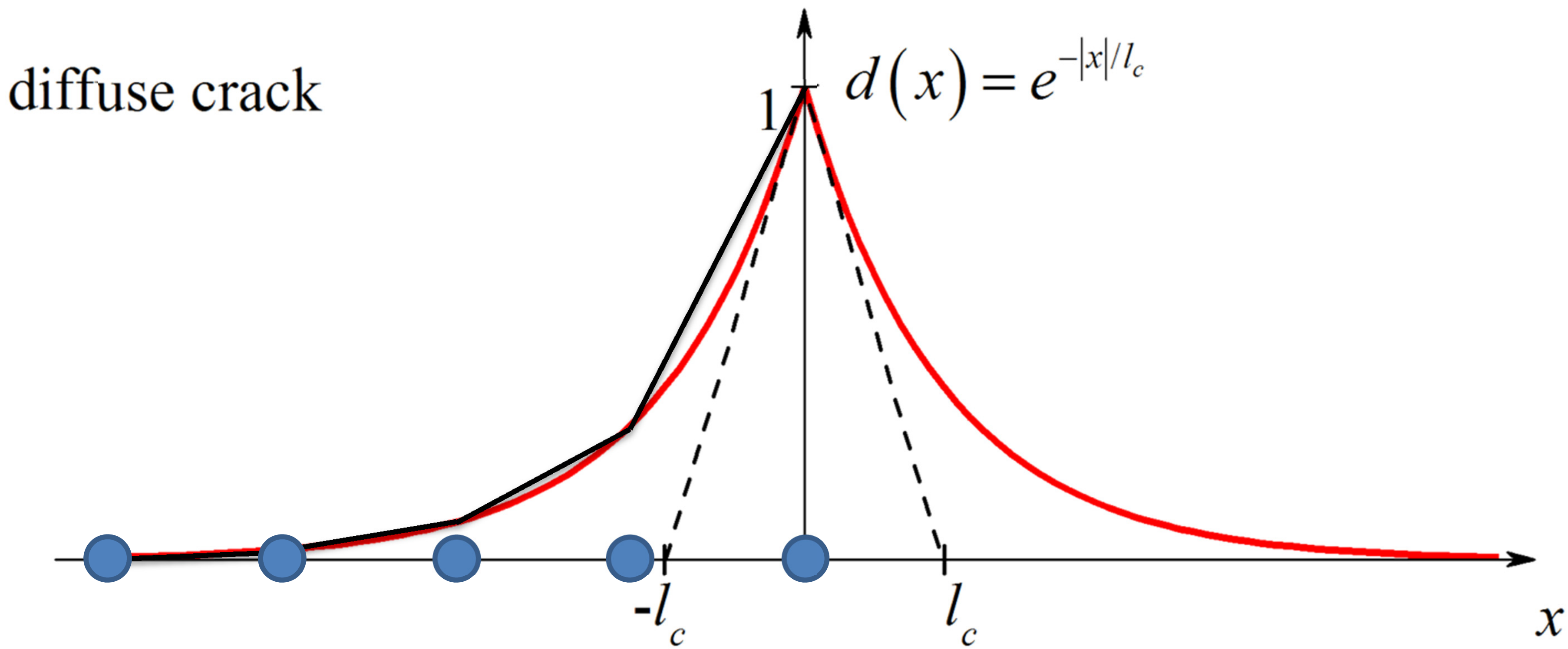
How fine should the mesh be?



Solving **fracture mechanics** problem with Partial Differential Equations (**PDE**s)

Parameters

How fine should the mesh be?

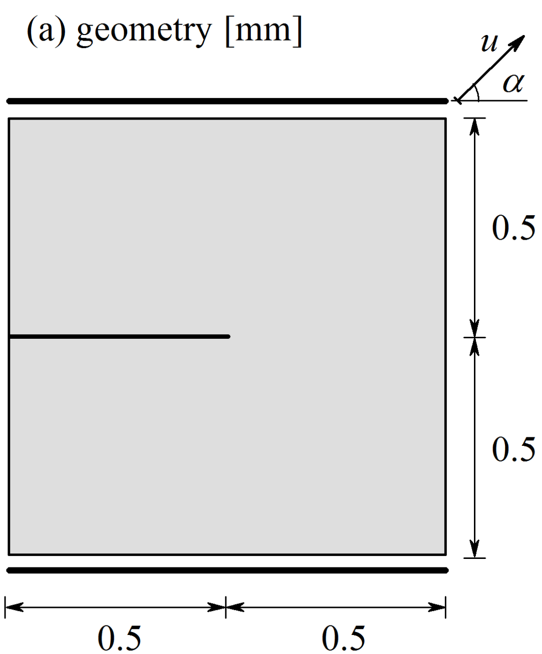


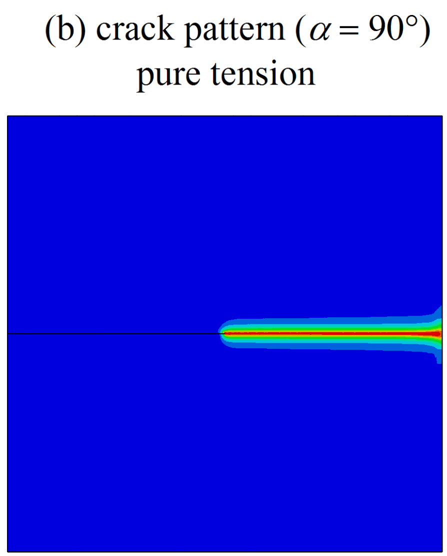
Solving **fracture mechanics** problem with Partial Differential Equations (**PDE**s)

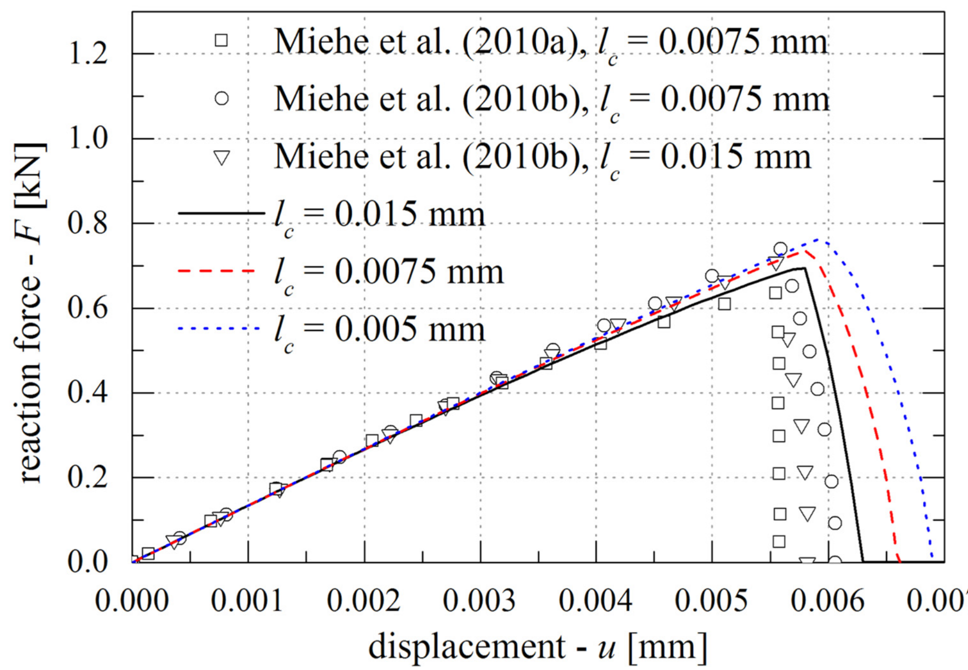
Molnár & Gravouil, 2017

Parameters

Single notched specimen under tension





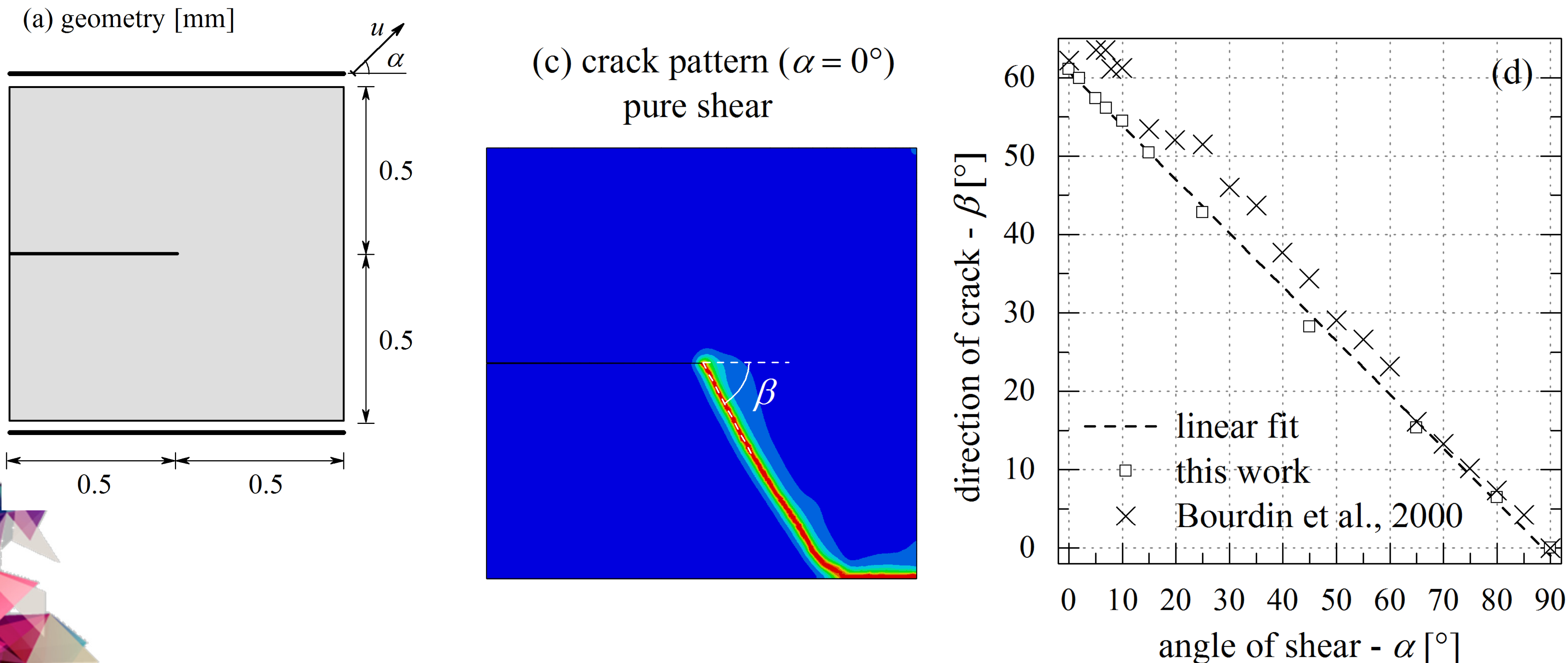


$$\Delta l_c = 300\% \rightarrow \Delta F_{\max} \approx 10\%$$

Effect of **length-scale**

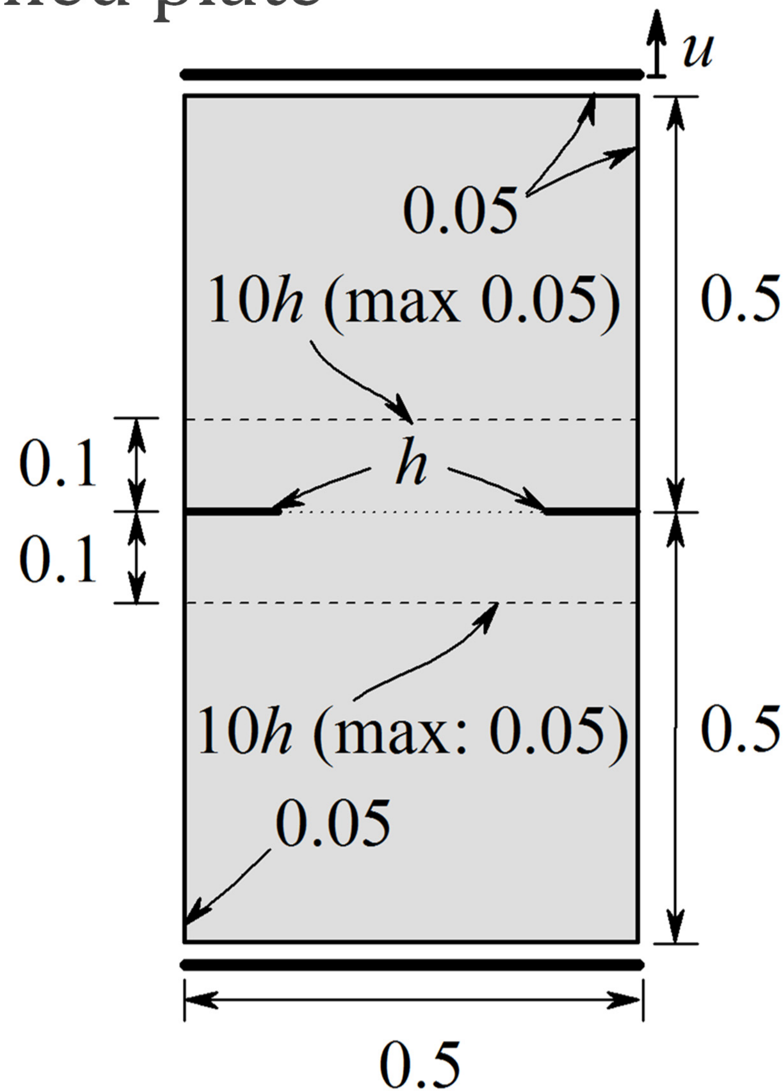
Parameters

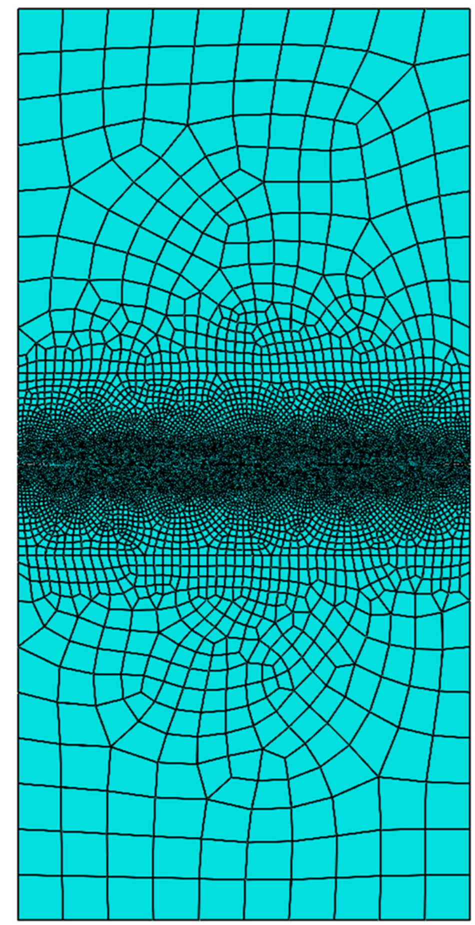
Single notched specimen under shear



Parameters

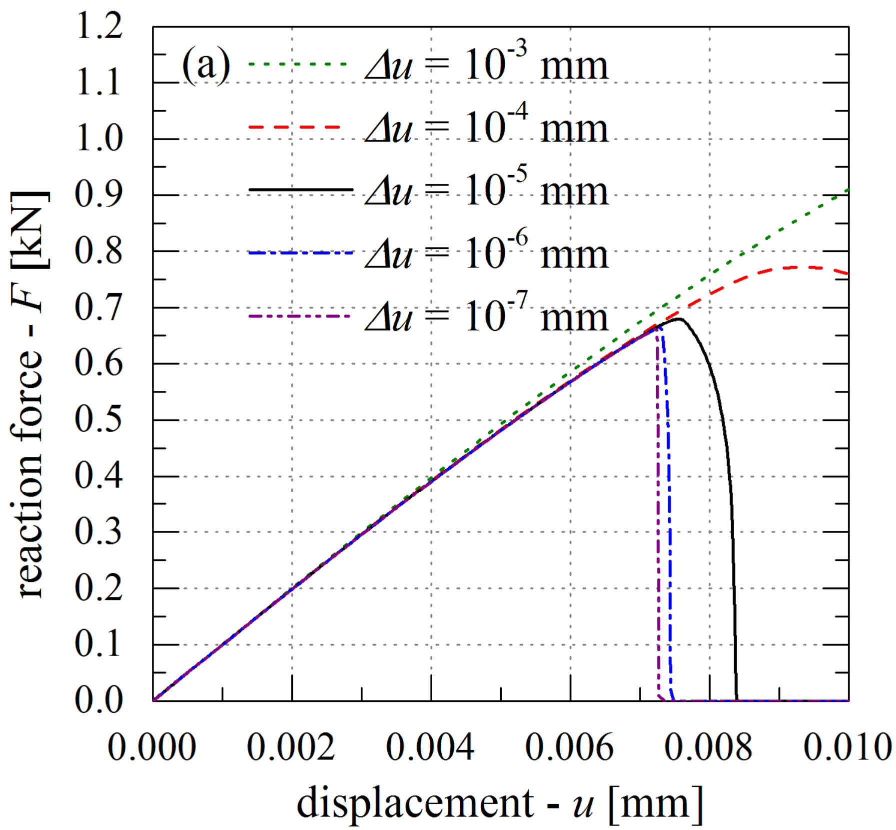
Double notched plate

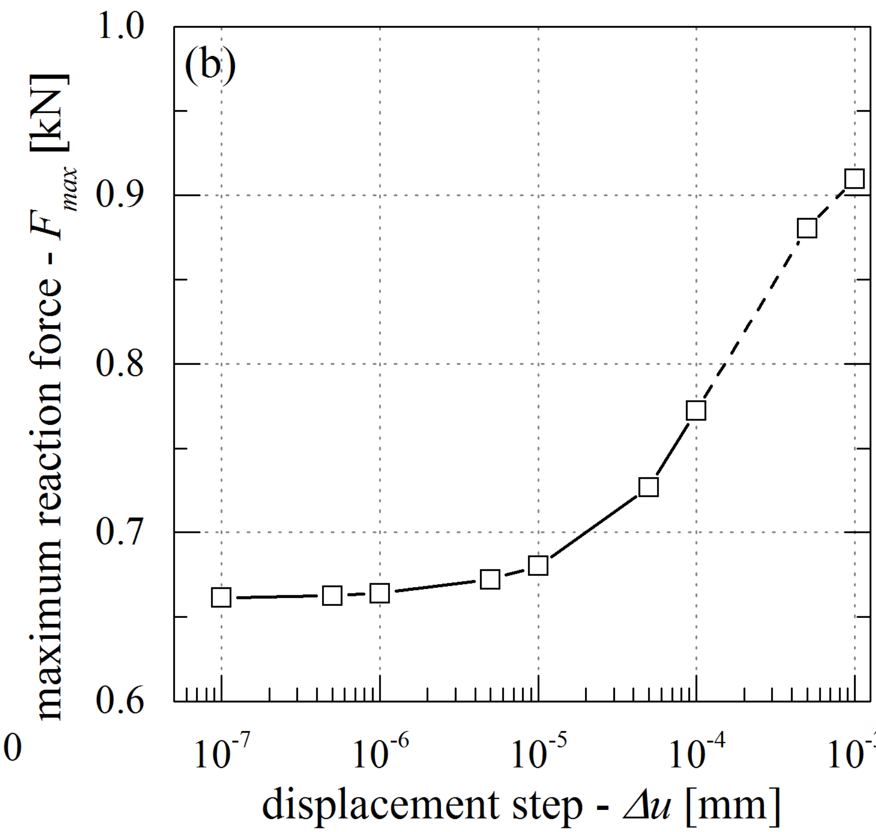




Parameters

Time step



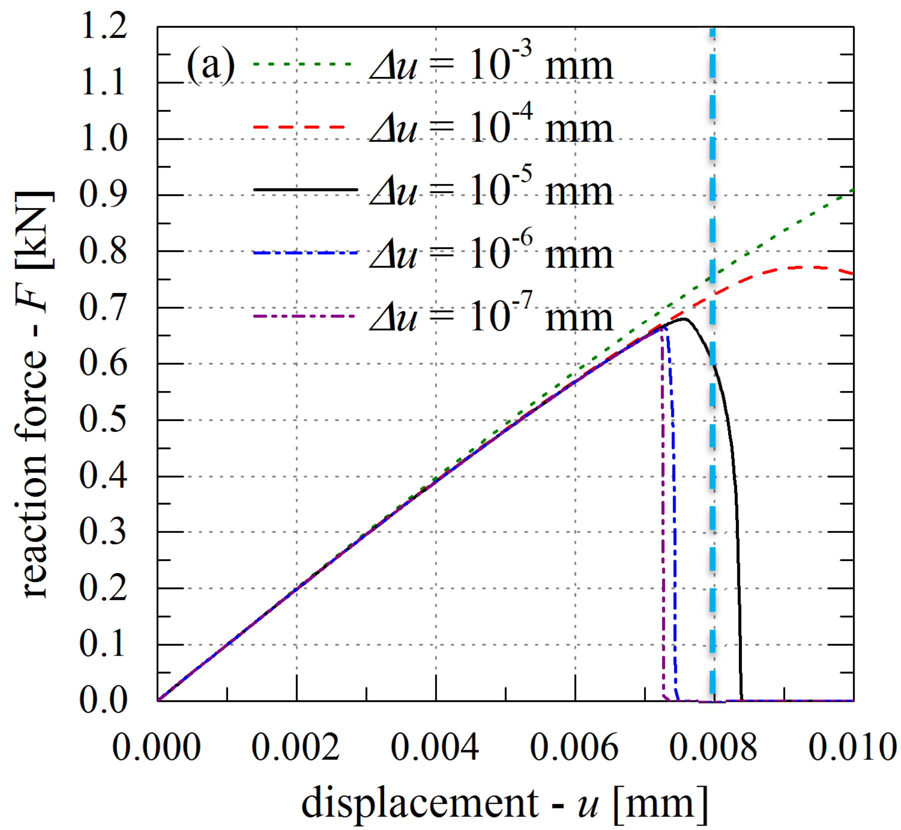


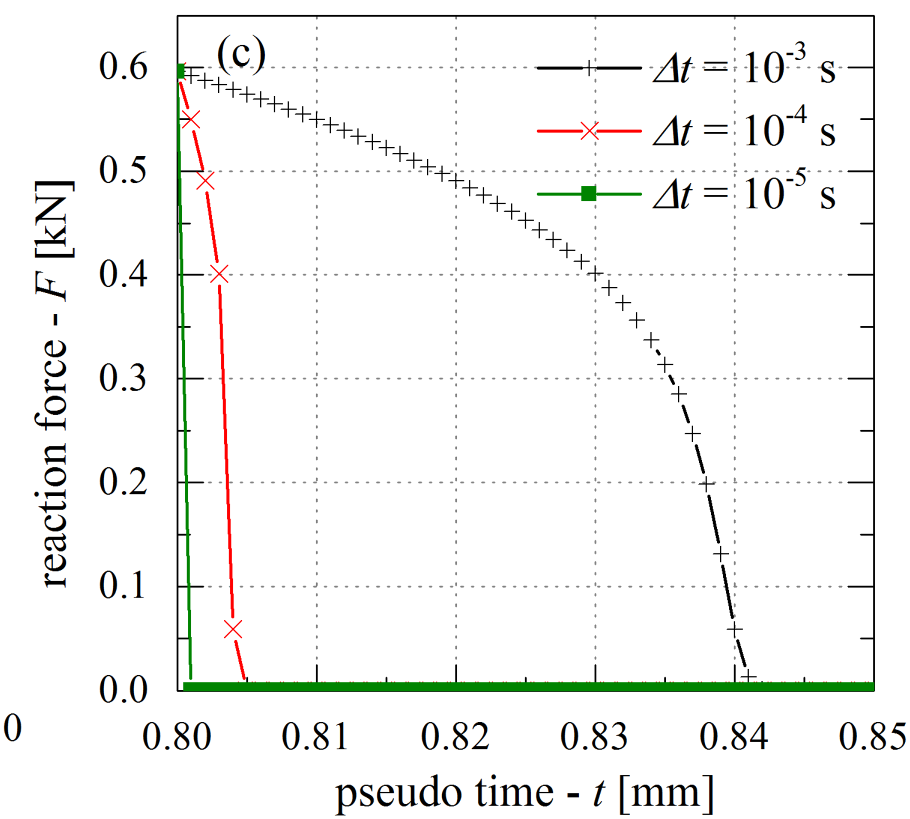
Molnár & Gravouil, 2017

Parameters

Time step

Deformation is applied until 0.008 mm then stopped





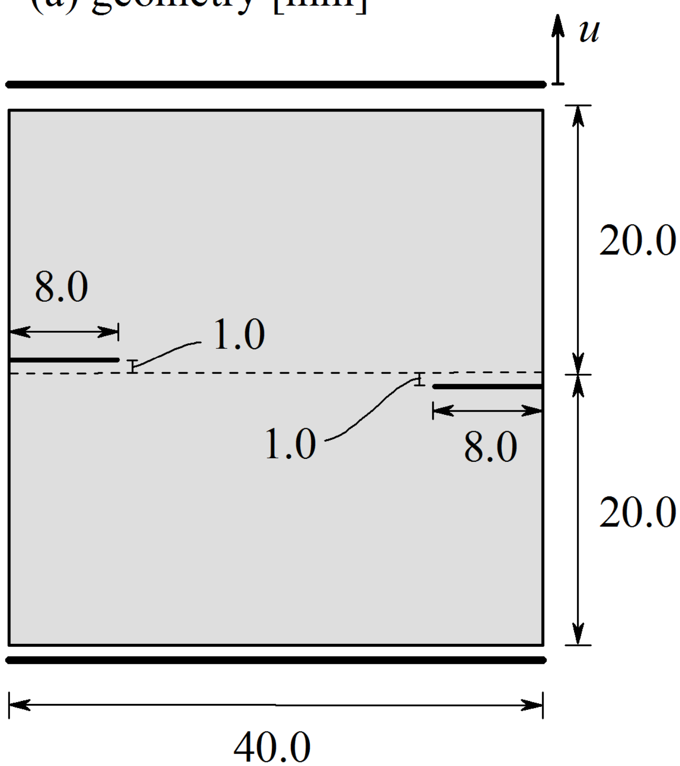
For details see ***Tutorial 3: Cracked cylinder in tension*** on www.molnar-research.com

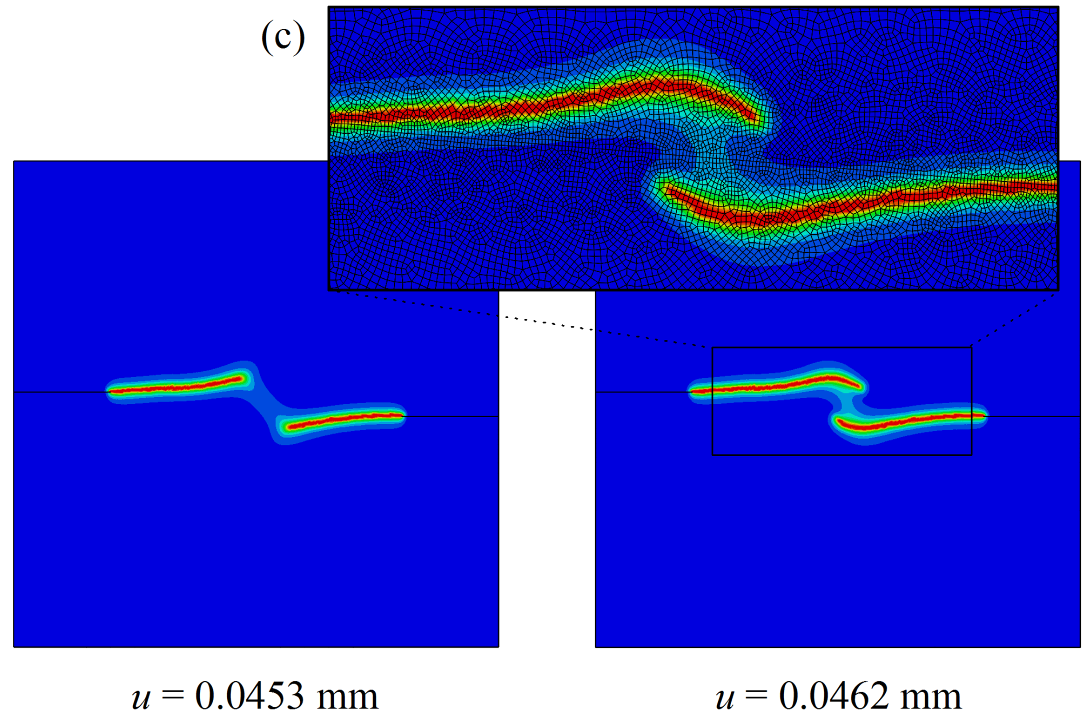
Molnár & Gravouil, 2017

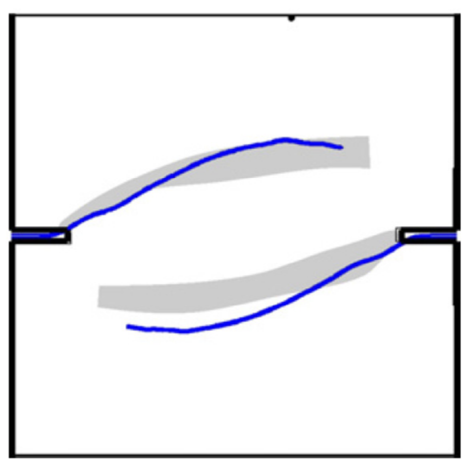
Examples

Asymmetric double notched plate

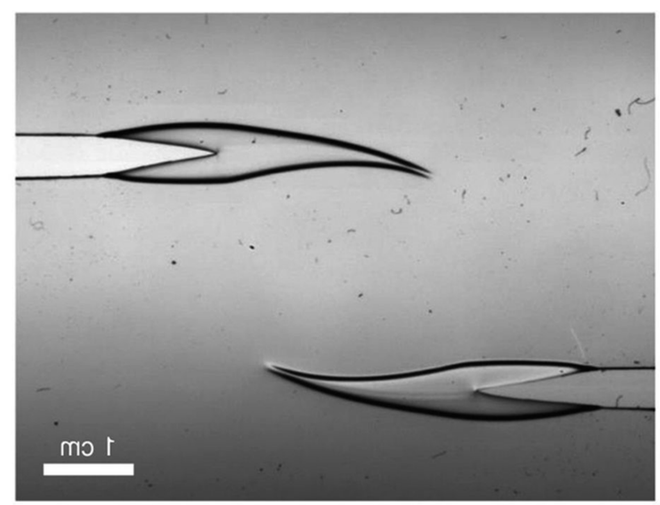
(a) geometry [mm]







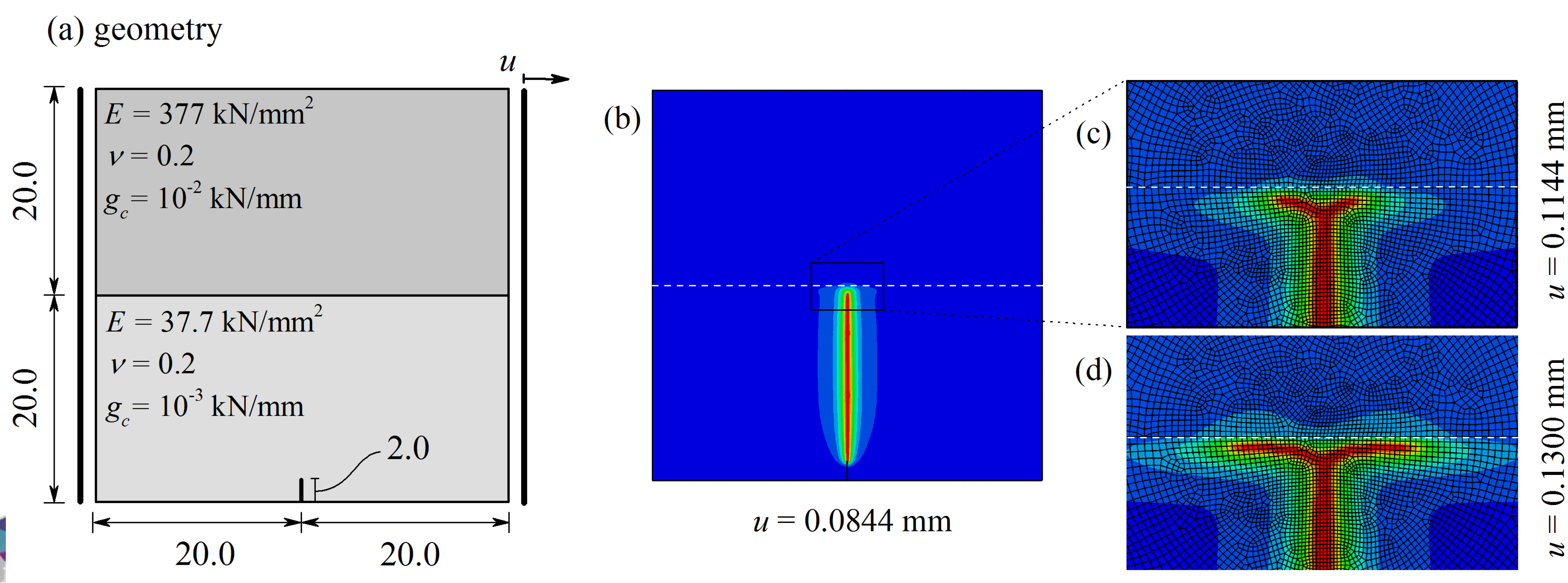
Réthoré et al., 2010



Koivisto et al., 2016

Examples

Bi-material tension

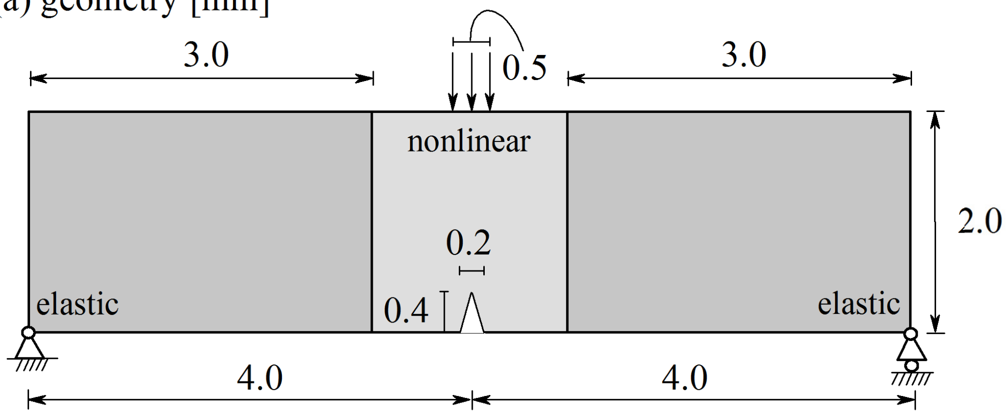


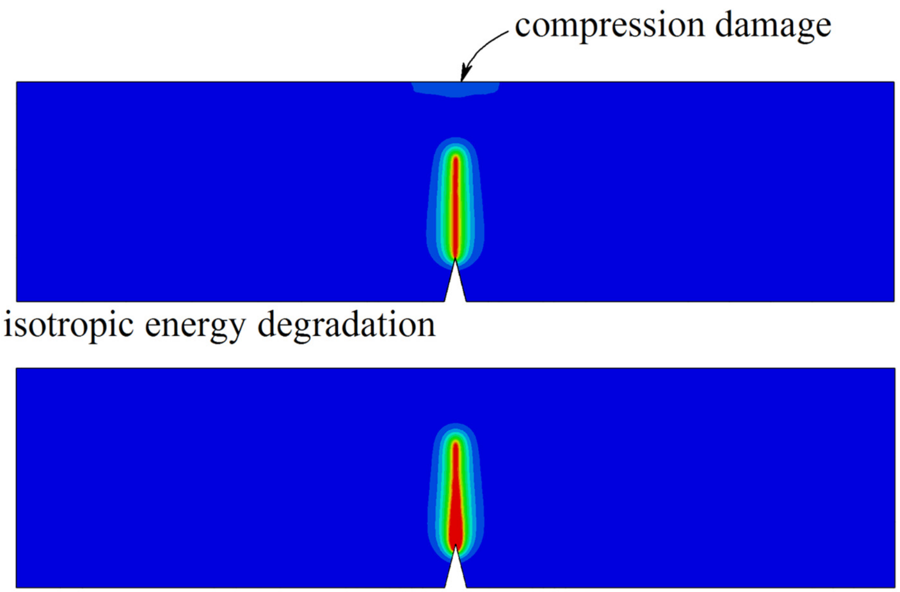
Molnár & Gravouil, 2017

Examples

Positive and negative energy degradation

(a) geometry [mm]





isotropic energy degradation

anisotropic energy degradation

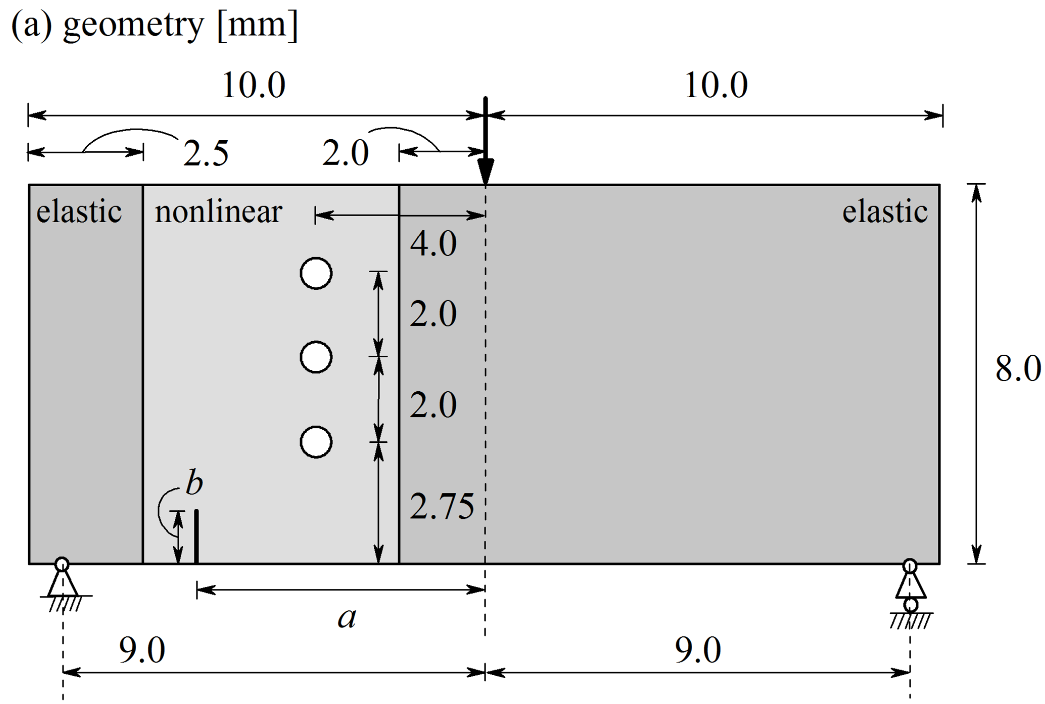
$$\psi_0^{\pm} = \frac{E\nu}{(1+\nu)(1-2\nu)} \langle tr(\boldsymbol{\varepsilon}) \rangle_{\pm}^2 + \frac{E}{2(1+\nu)} \left(\langle \varepsilon_2 \rangle_{\pm}^2 + \langle \varepsilon_2 \rangle_{\pm}^2 \right)$$

$$\psi = \psi_0^+ g(d) + \psi_0^-$$

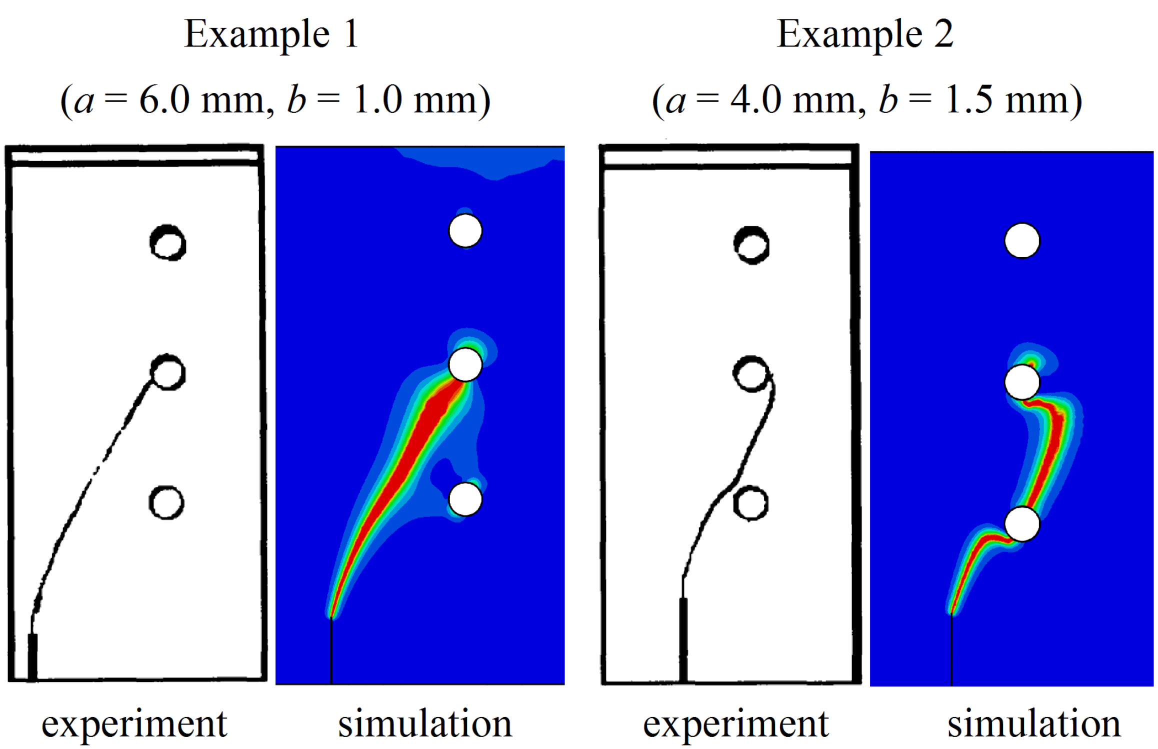
Molnár & Gravouil, 2017

Examples

Asymmetric bending

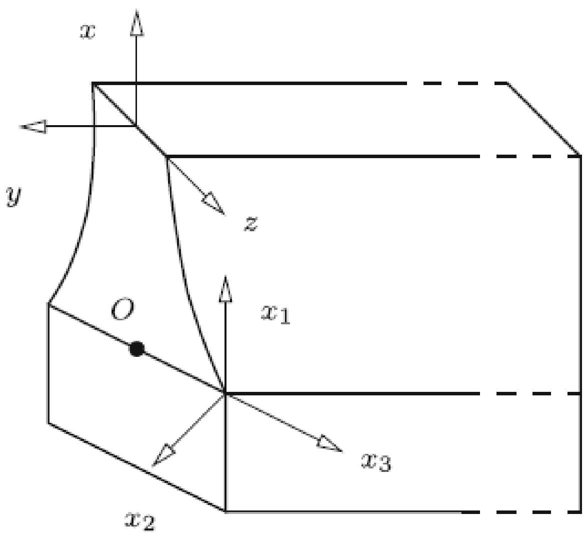


Bittencourt et al., 1996

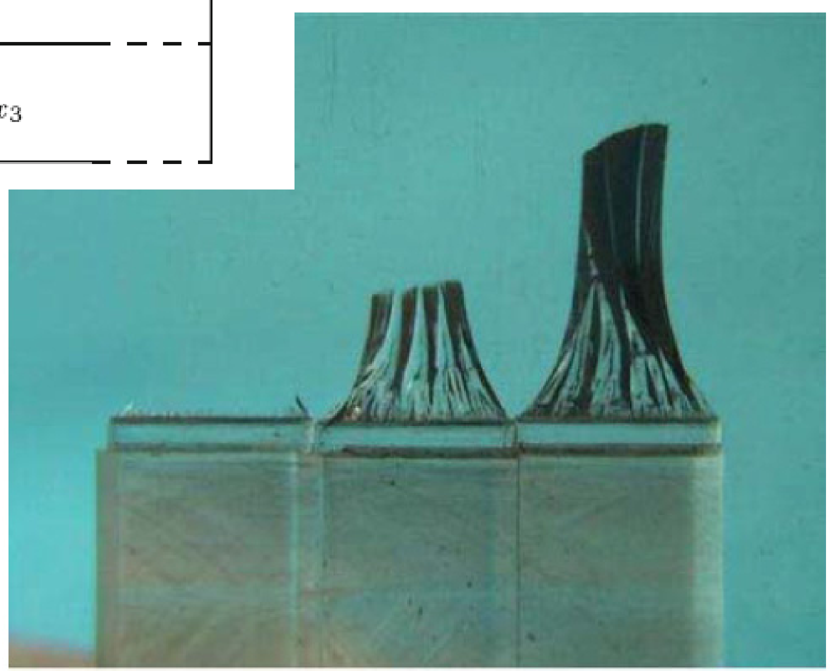


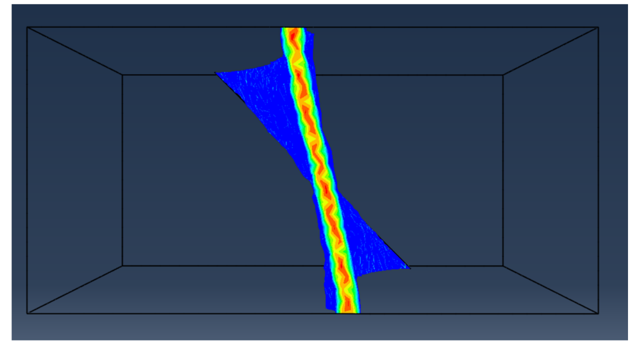
3D Examples

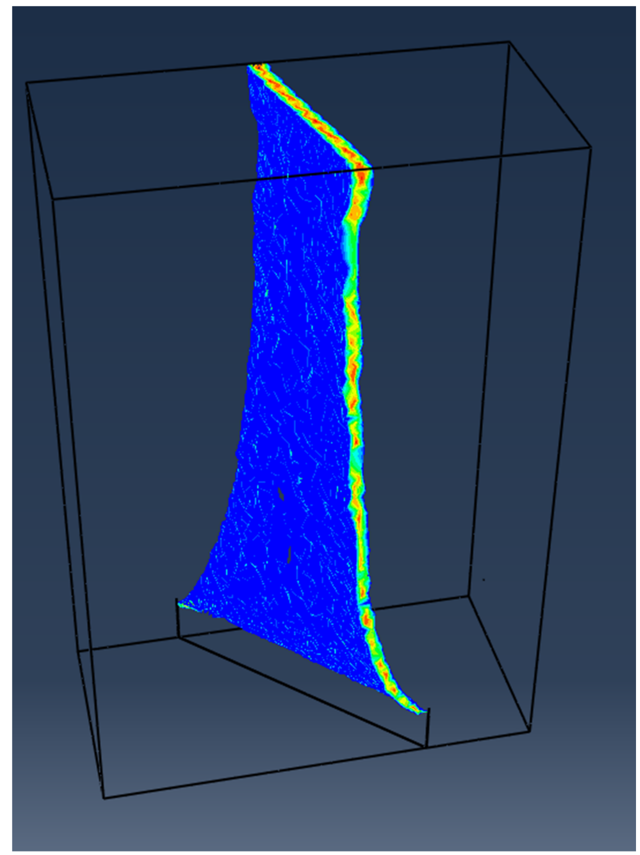
Inclined crack in bending



Lazarus et al., 2008



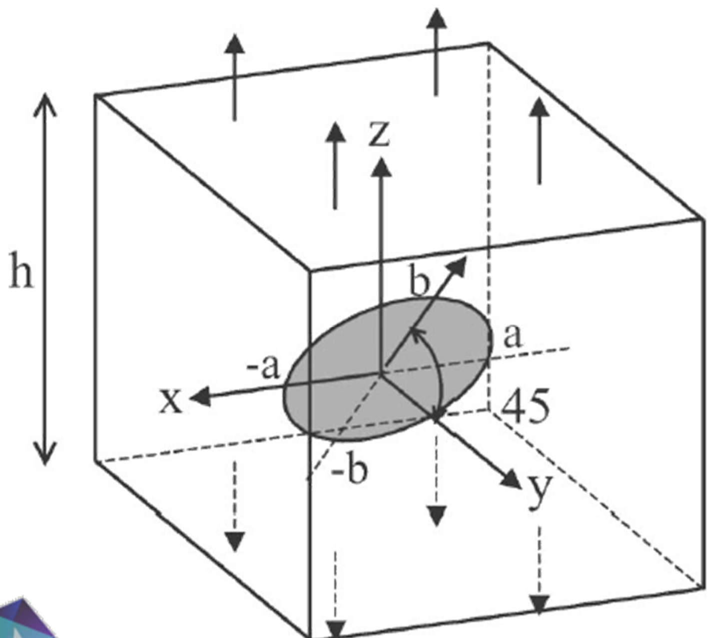




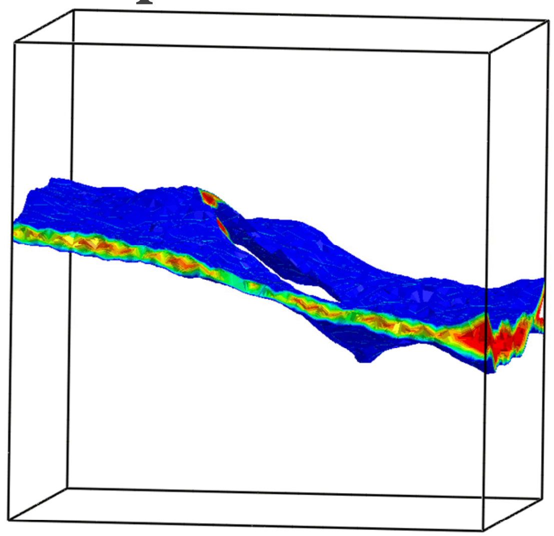
Molnár & Gravouil, 2017

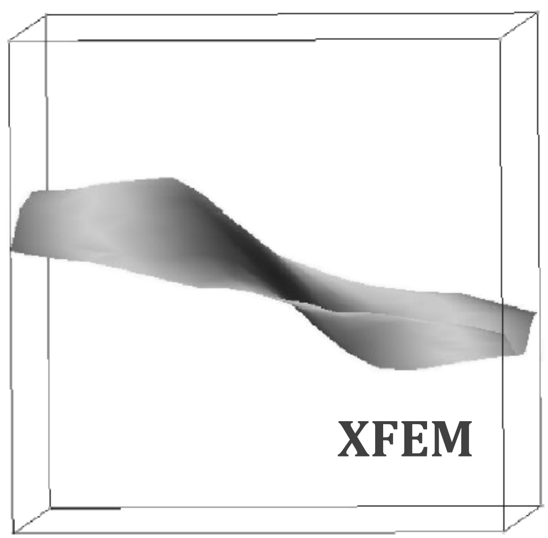
3D Examples

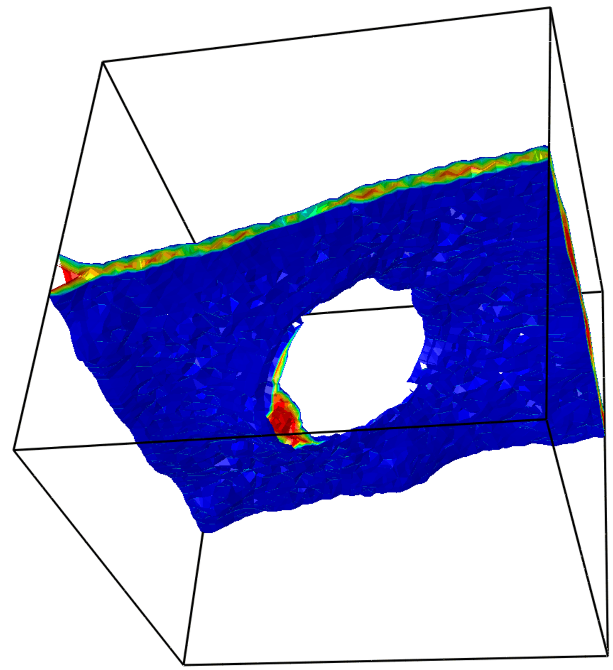
Inclined penny shape crack in tension



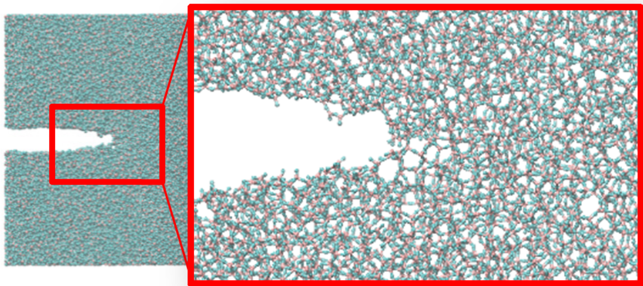
Gravouil et al., 2002



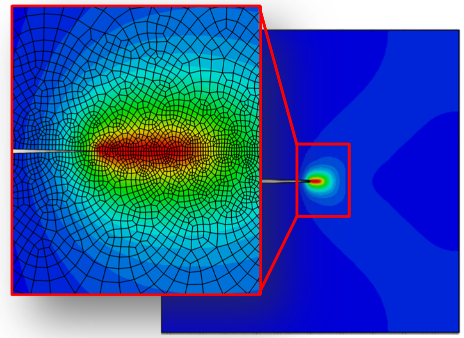




Conclusion



Microstructure



Phase-field

Hybrid FDEM

XFEM/GFEM

Cohesive Zones

Predefined crack

Advantages and disadvantages

- crack initiation, propagation
- branching, merging
- fixed mesh
- fully 3D

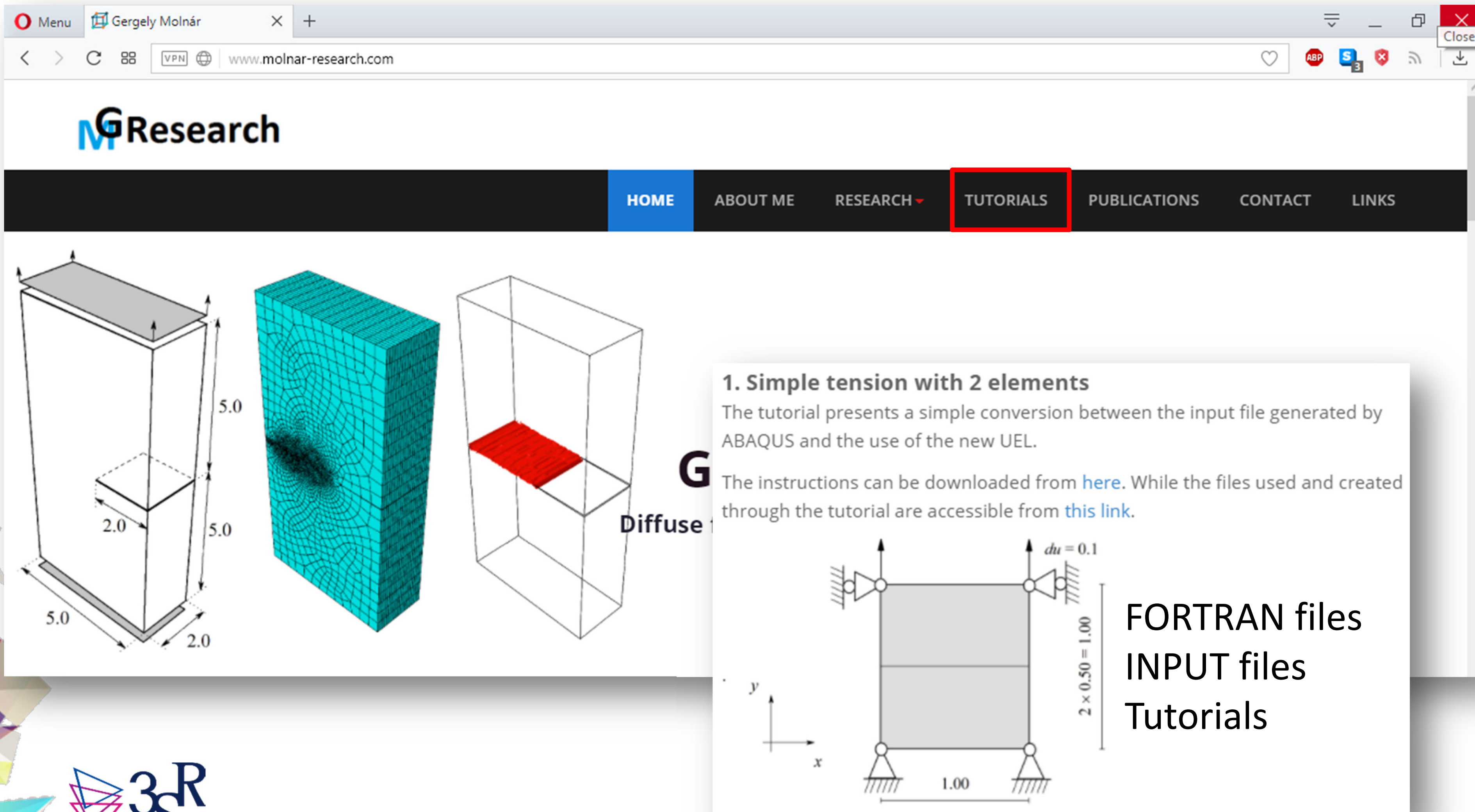
- fine mesh
- finite crack size
- efficiency/robustness

Versatility

dynamics, shells, nonlinear elasticity, large strains, coupled problems, plasticity, anisotropy, etc…

Where to find it?

Examples and tutorials: www.molnar-research.com



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J.-D. Wörner, Glasbau, 2001.

Thank you for your attention

Now let's try it out!

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3SR